Illustrative Mathematics

Algebra 1
Introduction to Quadratics

Lessons 1-5

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Illustrative Mathematics

Lesson 1: A Different Kind of Change

1.1: Notice and Wonder: Three Tables

Look at the patterns in the 3 tables. What do you notice? What do you wonder?

x	y
1	0
2	5
3	10
4	15
5	20

x	y
1	3
2	6
3	12
4	24
5	48

x	y
1	8
2	11
3	10
4	5
5	-4

1.2: Measuring a Garden

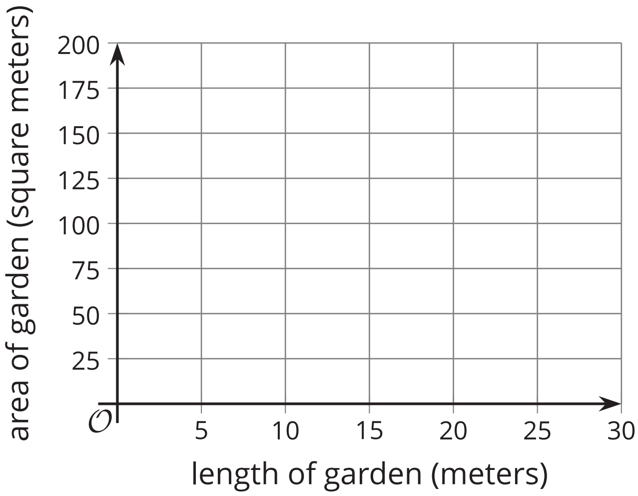
Noah has 50 meters of fencing to completely enclose a rectangular garden in the backyard.

1. Draw some possible diagrams of Noah's garden. Label the length and width of each rectangle.

2. Find the length and width of such a rectangle that would produce the largest possible area. Explain or show why you think that pair of length and width gives the largest possible area.

1.3: Plotting the Measurements of the Garden

1. Plot some values for the length and area of the garden on the coordinate plane.



2. What do you notice about the plotted points?

3. The points $(3, 66)$ and $(22, 66)$ each represent the length and area of the garden. Plot these 2 points on coordinate plane, if you haven't already done so. What do these points mean in this situation?

4. Could the point $(1, 25)$ represent the length and area of the garden? Explain how you know.

Are you ready for more?

1. What happens to the area when you interchange the length and width? For example, compare the areas of a rectangle of length 11 meters and width 14 meters with a rectangle of length 14 meters and width 11 meters.

2. What patterns would you notice if you were to plot more length and area pairs on the graph?

Lesson 1 Summary

In this lesson, we looked at the relationship between the side length and the area of a rectangle when the perimeter is unchanged.

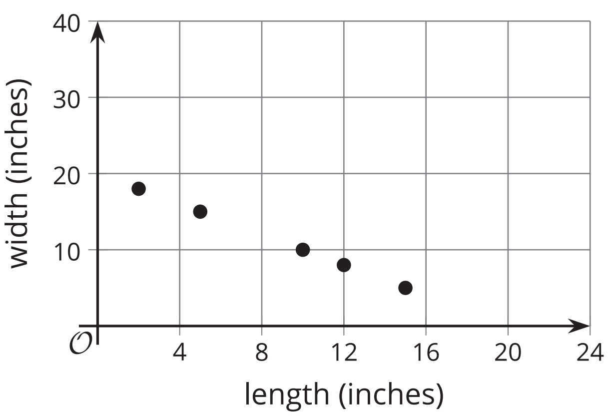
If a rectangle has a perimeter of 40 inches, we can represent the possible lengths and widths as shown in the table.

length (inches)	width (inches)
2	18
5	15
10	10
12	8
15	5

We know that twice the length and twice the width must equal 40, which means that the length plus width must equal 20, or $\ell + w = 20$.

To find the width given a length ℓ, we can write: $w = 20 - \ell$.

The relationship between the length and the width is linear. If we plot the points from the table representing the length and the width, they form a line.

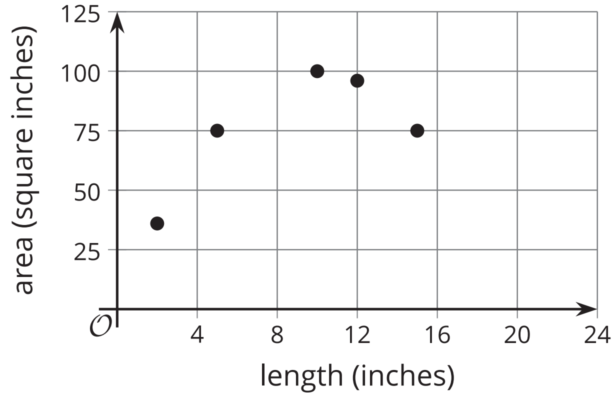


What about the relationship between the side lengths and the area of rectangles with perimeter of 40 inches?

Here are some possible areas of different rectangles whose perimeter are all 40 inches.

length (inches)	width (inches)	area (square inches)
2	18	36
5	15	75
10	10	100
12	8	96
15	5	75

Here is a graph of the lengths and areas from the table:



Notice that, initially, as the length of the rectangle increases (for example, from 5 to 10 inches), the area also increases (from 75 to 100 square inches). Later, however, as the length increases (for example, from 12 to 15), the area decreases (from 96 to 75).

We have not studied relationships like this yet and will investigate them further in this unit.

Lesson 1: A Different Kind of Change

Cool Down: 100 Meters of Fencing

A rectangular yard is enclosed by 100 meters of fencing. The table shows some possible values for the length and width of the yard.

length (meters)	width (meters)	area (square meters)
10	40	400
20	30	
25	25	625
35	15	525
40		

1. Complete the table with the missing values.

2. If the values for length and area are plotted, what would the graph look like?

3. How is the relationship between the length and the area of the rectangle different from other kinds of relationships we've seen before?

Unit 6 Lesson 1 Cumulative Practice Problems

1. Here are a few pairs of positive numbers whose sum is 50.

   a. Find the product of each pair of numbers.

   b. Find a pair of numbers that have a sum of 50 and will produce the largest possible product.

   c. Explain how you determined which pair of numbers have the largest product.

first number	second number	product
1	49	
2	48	
10	40	

2. Here are some lengths and widths of a rectangle whose perimeter is 20 meters.

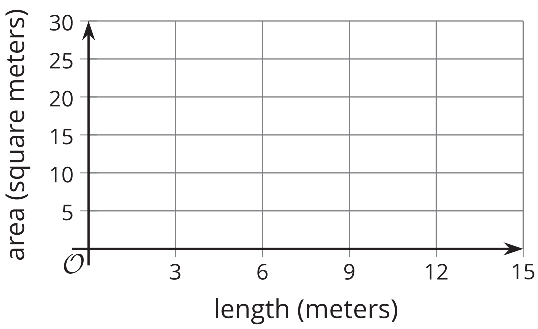
   a. Complete the table. What do you notice about the areas?

length (meters)	width (meters)	area (square meters)
1	9	
3	7	
5		
7		
9		

   b. Without calculating, predict whether the area of the rectangle will be greater or less than 25 square meters if the length is 5.25 meters.

   c. On the coordinate plane, plot the points for length and area from your table.

   Do the values change in a linear way? Do they change in an exponential way?



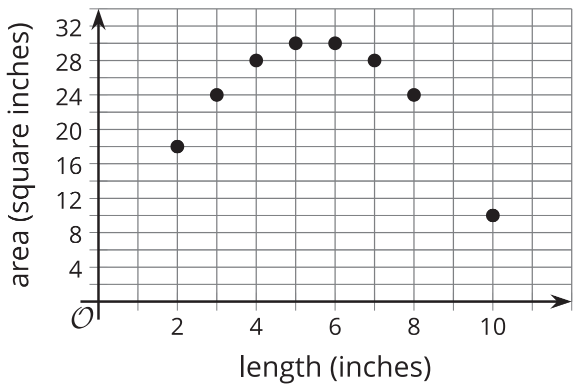
3. The table shows the relationship between x and y, the side lengths of a rectangle, and the area of the rectangle.

    a. Explain why the relationship between the side lengths is linear.

    b. Explain why the relationship between x and the area is neither linear nor exponential.

x (cm)	y (cm)	area (sq cm)
2	4	8
4	8	32
6	12	72
8	16	128

4. Which statement best describes the relationship between a rectangle's side length and area as represented by the graph.



    A. As the side length increases by 1, the area increases and then decreases by an equal amount.

    B. As the side length increases by 1, the area increases and then decreases by an equal factor.

    C. As the side length increases by 1, the area does not increase or decrease by an equal amount.

    D. As the side length increases by 1, the area does not change.

5. Copies of a book are arranged in a stack. Each copy of a book is 2.1 cm thick.

   a. Complete the table.

   b. What do you notice about the differences in the height of the stack of books when a new copy of the book is added?

   c. What do you notice about the factor by which the height of the stack of books changes when a new copy is added?

copies of book	stack height in cm
0	
1	
2	
3	
4	

   d. How high is a stack of b books?

(From Unit 5, Lesson 2.)

6. The value of a phone when it was purchased was \$500. It loses $\frac{1}{5}$ of its value a year.

   a. What is the value of the phone after 1 year? What about after 2 years? 3 years?

   b. Tyler says that the value of the phone decreases by \$100 each year since $\frac{1}{5}$ of 500 is 100. Do you agree with Tyler? Explain your reasoning.

(From Unit 5, Lesson 4.)

7. *Technology required*. The data in the table represents the price of one gallon of milk in different years.

Use graphing technology to create a scatter plot of the data.

x, time (years)	price per gallon of milk (dollars)
1930	0.26
1935	0.47
1940	0.52
1940	0.50
1945	0.63
1950	0.83
1955	0.93
1960	1.00
1965	1.05
1970	1.32
1970	1.25
1975	1.57
1985	2.20
1995	2.50
2005	3.20
2018	2.90
2018	3.25

a. Does a linear model seem appropriate for this data? Why or why not?

b. If the data seems appropriate, create the line of best fit. Round to two decimal places.

c. What is the slope of the line of best fit, and what does it mean in this context? Is it realistic?

d. What is the y-intercept of the line of best fit, and what does it mean in this context? Is it realistic?

(From Unit 3, Lesson 5.)

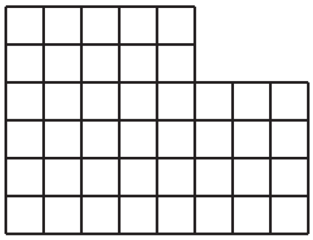
8. Give a value for r that indicates that a line of best fit has a negative slope and models the data well.

(From Unit 3, Lesson 7.)

Lesson 2: How Does it Change?

2.1: Squares in a Figure

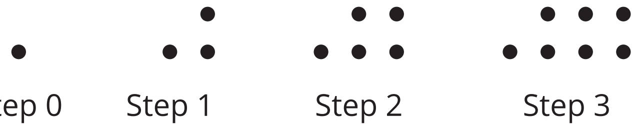
How does each expression represent the number of small squares in the figure?



- Expression A: $6 \cdot 8 - 2 \cdot 3$
- Expression B: $4 \cdot 8 + 2 \cdot 5$
- Expression C: $8 + 8 + 8 + 8 + 5 + 5$
- Expression D: $5 \cdot 6 + 3 \cdot 4$

2.2: Patterns of Dots

Pattern 1



Pattern 2

1. Study the 2 patterns of dots.
    a. How are the number of dots in each pattern changing?

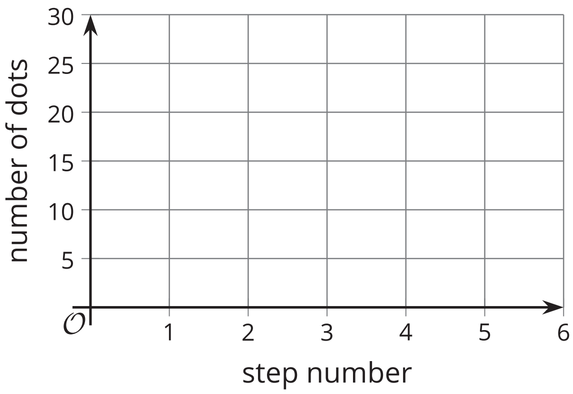
    b. How would you find the number of dots in the 5th step in each pattern?

2. Complete the table with the number of dots in each pattern.

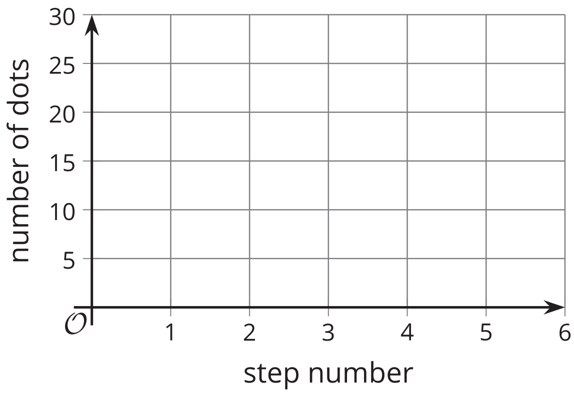
step	number of dots in Pattern 1	number of dots in Pattern 2
0		
1		
2		
3		
4		
5		
10		
n		

3. Plot the number of dots at each step number.

Pattern 1

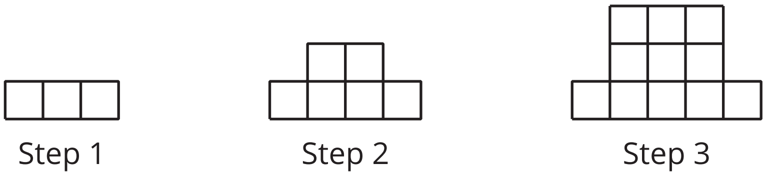


Pattern 2



4. Explain why the graphs of the 2 patterns look the way they do.

2.3: Expressing a Growth Pattern



Here is a pattern of squares.

1. Is the number of small squares growing linearly? Explain how you know.

2. Complete the table.

step	number of small squares
1	
2	
3	
4	
5	
10	
12	
n	

3. Is the number of small squares growing exponentially? Explain how you know.

Are you ready for more?

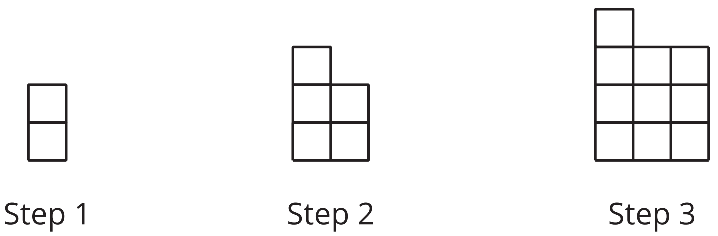
Han wrote $n(n+2)-2(n-1)$ for the number of small squares in the design at Step n.

1. Explain why Han is correct.

2. Label the picture in a way that shows how Han saw the pattern when writing his expression.

Lesson 2 Summary

In this lesson, we saw some quantities that change in a particular way, but the change is neither linear nor exponential. Here is a pattern of shapes, followed by a table showing the relationship between the step number and the number of small squares.



step	total number of small squares
1	2
2	5
3	10
n	n^2+1

The number of small squares increases by 3, and then by 5, so we know that the growth is not linear. It is also not exponential because it is not changing by the same factor each time. From Step 1 to Step 2, the number of small squares grows by a factor of $\frac{5}{2}$, while from Step 2 to Step 3, it grows by a factor of 2.

From the diagram, we can see that in Step 2, there is a 2-by-2 square plus 1 small square added on top. Likewise, in Step 3, there is a 3-by-3 square with 1 small square added. We can reason that the nth step is an n-by-n arrangement of small squares with an additional small square on top, giving the expression $n^2 + 1$ for the number of small squares.

The relationship between the step number and the number of small squares is a quadratic relationship, because it is given by the expression $n^2 + 1$, which is an example of a **quadratic expression**. We will investigate quadratic expressions in depth in future lessons.

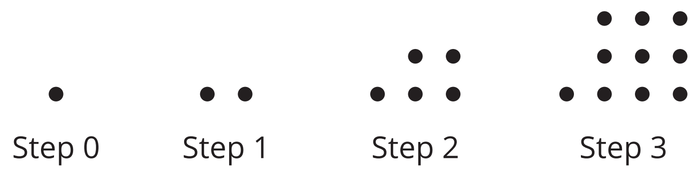
Lesson 2: How Does it Change?

Cool Down: Comparing Types of Growth

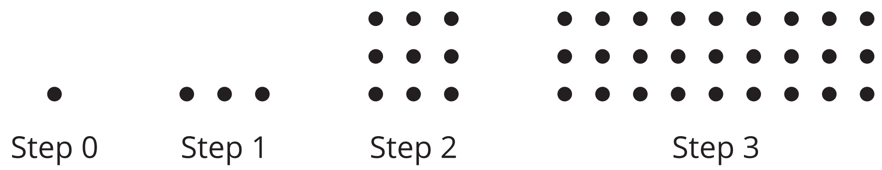
Here are three patterns of dots.

Pattern A

Pattern B



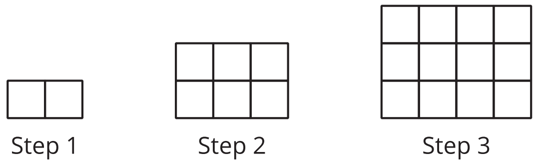
Pattern C



Which pattern shows a quadratic relationship between the step number and the number of dots? Explain or show how you know.

Unit 6 Lesson 2 Cumulative Practice Problems

1. How many small squares are in Step 10?



   A. 10

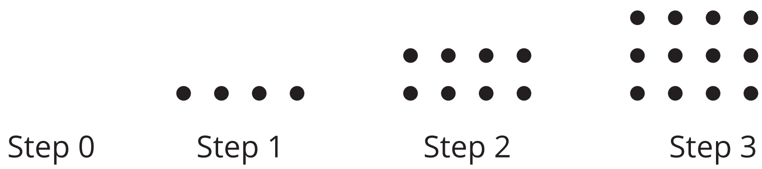
   B. 11

   C. 90

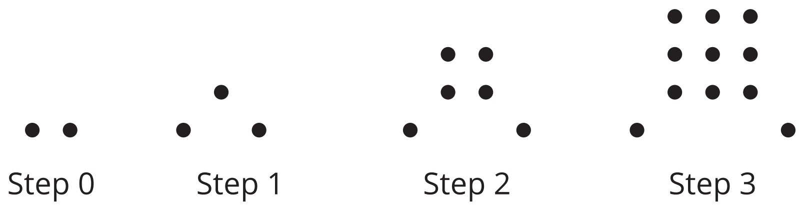
   D. 110

2. Here are 2 patterns of dots.

   Pattern A



   Pattern B



   a. How many dots will there be in Step 4 of each pattern?

   b. Which pattern shows a quadratic relationship between the step number and the number of dots? Explain how you know.

3. Here are descriptions for how two dot patterns are growing.

    - Pattern A: Step 2 has 10 dots. It grows by 3 dots at each additional step.
    - Pattern B: The total number of dots can be expressed by $2n^2 + 1$, where n is the step number.

    For each pattern, draw a diagram of Step 0 to Step 3.

4. Each expression represents the total number of dots in a pattern where n represents the step number.

    Select **all** the expressions that represent a quadratic relationship between the step number and the total number of dots. (If you get stuck, consider sketching the first few steps of each pattern as described by the expression.)

    A. n^2

    B. $2n$

    C. $n \cdot n$

    D. $n + n$

    E. $n + 2$

    F. $n \div 2$

5. The function C gives the percentage of homes using only cell phone service x years after 2004. Explain the meaning of each statement.

   a. $C(10) = 35$

   b. $C(x) = 10$

   c. How is $C(10)$ different from $C(x) = 10$?

(From Unit 4, Lesson 3.)

6. Here are some lengths, widths, and areas of a garden whose perimeter is 40 feet.

   a. Complete the table with the missing measurements.

   b. What lengths and widths do you think will produce the largest possible area? Explain how you know.

length (ft)	width (ft)	area (sq ft)
4	16	64
8	12	
10		
12		96
14		
16		64

(From Unit 6, Lesson 1.)

7. A bacteria population is 10,000 when it is first measured and then doubles each day.

   a. Use this information to complete the table.

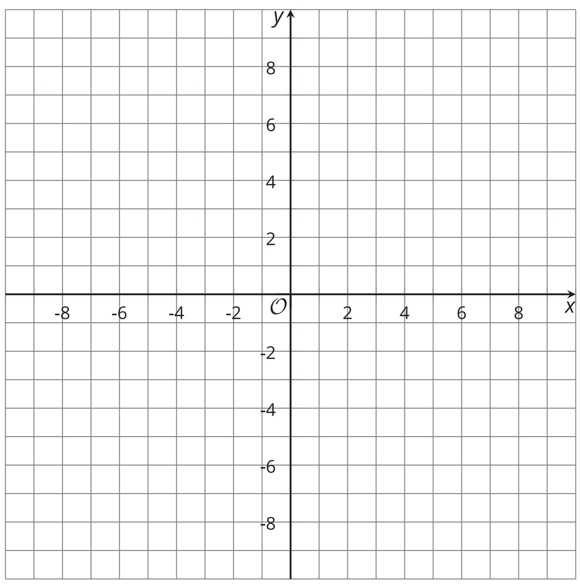
   b. Which is the first day, after the antibiotic is given, that the bacteria population is more than 1,000,000?

   c. Write an equation relating p, the bacteria population, to d, the number of days since it was first measured.

d, time (days)	p, population (thousands)
0	
1	
2	
5	
10	
d	

(From Unit 5, Lesson 3.)

8. Graph the solutions to the inequality $7x - 3y \geq 21$.



(From Unit 2, Lesson 21.)

Lesson 3: Building Quadratic Functions from Geometric Patterns

3.1: Quadratic Expressions and Area

Figure A is a large square. Figure B is a large square with a smaller square removed. Figure C is composed of two large squares with one smaller square added.

Figure A

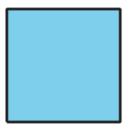


Figure B

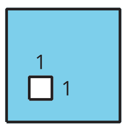
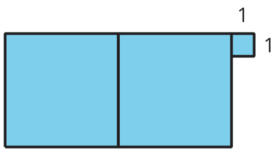


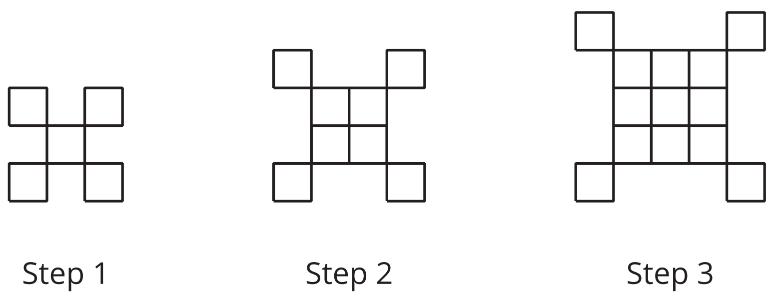
Figure C



Write an expression to represent the area of each shaded figure when the side length of the large square is as shown in the first column.

side length of large square	area of A	area of B	area of C
4			
x			
$4x$			
$(x+3)$			

3.2: Expanding Squares



1. If the pattern continues, what will we see in Step 5 and Step 18?

a. Sketch or describe the figure in each of these steps.

b. How many small squares are in each of these steps? Explain how you know.

2. Write an equation to represent the relationship between the step number n and the number of squares y. Be prepared to explain how each part of your equation relates to the pattern. (If you get stuck, try making a table.)

3. Sketch the first 3 steps of a pattern that can be represented by the equation $y = n^2 - 1$.

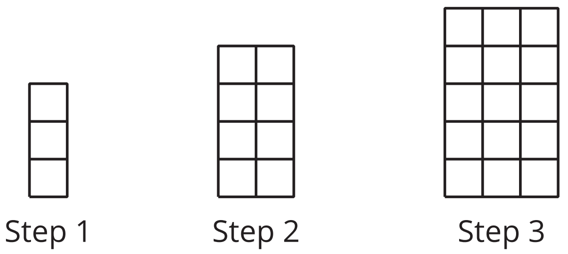
Are you ready for more?

1. For the original step pattern in the statement, write an equation to represent the relationship between the step number n and the perimeter, P.

2. For the step pattern you created in part 3 of the activity, write an equation to represent the relationship between the step number n and the perimeter, P.

3. Are these linear functions?

3.3: Growing Steps



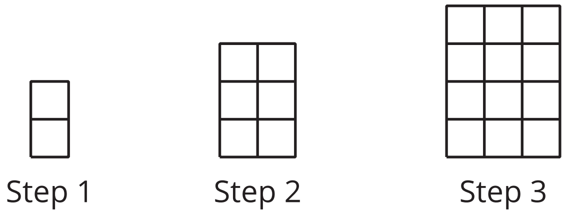
1. Sketch the next step in the pattern.

2. Kiran says that the pattern is growing linearly because as the step number goes up by 1, the number of rows and the number of columns also increase by 1. Do you agree? Explain your reasoning.

3. To represent the number of squares after n steps, Diego and Jada wrote different equations. Diego wrote the equation $f(n) = n(n + 2)$. Jada wrote the equation $f(n) = n^2 + 2n$. Are either Diego or Jada correct? Explain your reasoning.

Lesson 3 Summary

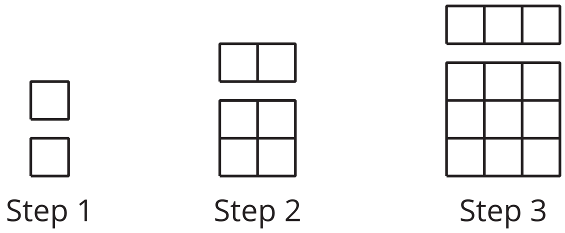
Sometimes a quadratic relationship can be expressed without using a squared term. Let's take this pattern of squares, for example.



From the first 3 steps, we can see that both the length and the width of the rectangle increase by 1 at each step. Step 1 is a 1-by-2 rectangle, Step 2 is a 2-by-3 rectangle, and Step 3 is a 3-by-4 rectangle. This suggests that Step n is a rectangle with side lengths n and $n+1$, so the number of squares at Step n is $n(n+1)$.

This expression may not look like quadratic expressions with a squared term, which we saw in earlier lessons, but if we apply the distributive property, we can see that $n(n+1)$ is equivalent to n^2+n.

We can also visually show that these expressions are the equivalent by breaking each rectangle into an n-by-n square (the n^2 in the expression) and an n-by-1 rectangle (the n in the expression).

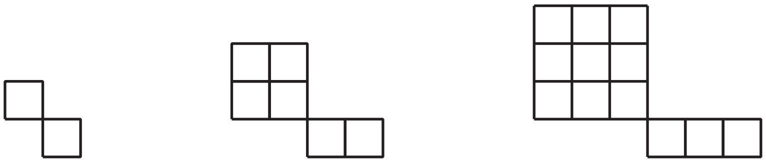


The relationship between the step number and the number of squares can be described by a **quadratic function** f whose input is n and whose output is the number of squares at Step n. We can define f with $f(n)=n(n+1)$ or $f(n)=n^2+n$.

Lesson 3: Building Quadratic Functions from Geometric Patterns

Cool Down: A Quadratic Function?

Here is a pattern of squares.



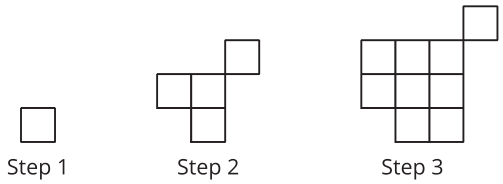
Step 1 Step 2 Step 3

1. Write an equation to represent the relationship between the step number and the number of squares in the pattern. Briefly describe how each part of the equation relates to the pattern.

2. Is the relationship between the step number and number of squares a quadratic function? Explain how you know.

Unit 6 Lesson 3 Cumulative Practice Problems

1. a. Sketch or describe the figure in Step 4 and Step 15.

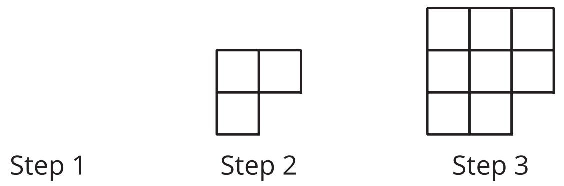


b. How many small squares will there be in each of these steps?

c. Write an equation to represent the relationship between the step number, n, and the number of small squares, y, in each step.

d. Explain how your equation relates to the pattern.

2. Which expression represents the relationship between the step number n and the total number of small squares in the pattern?



A. $n^2 + 1$

B. $n^2 - 1$

C. $n^2 - n$

D. $n^2 + n$

3. Each figure is composed of large squares and small squares. The side length of the large square is x. Write an expression for the area of the shaded part of each figure.

Figure A

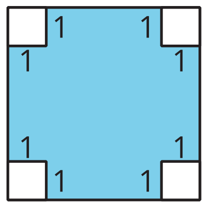
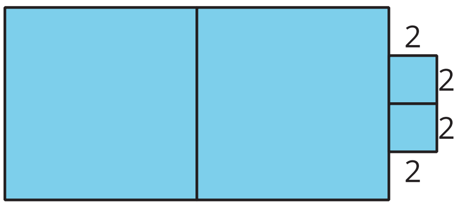


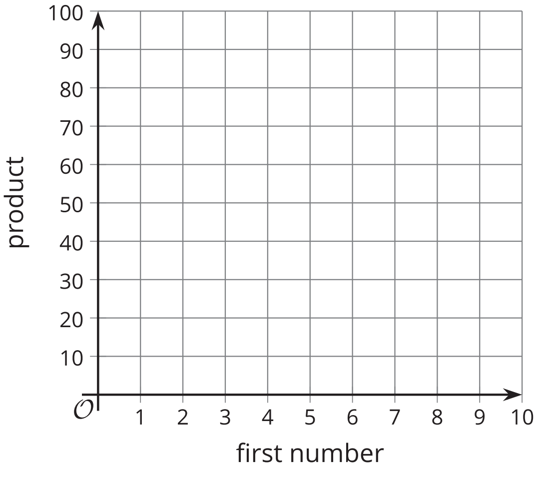
Figure B



4. Here are a few pairs of positive numbers whose difference is 5.

a. Find the product of each pair of numbers. Then, plot some points to show the relationship between the first number and the product.

first number	second number	product
1	6	
2	7	
3	8	
5	10	
7	12	



b. Is the relationship between the first number and the product exponential? Explain how you know.

(From Unit 6, Lesson 1.)

5. Here are some lengths and widths of a rectangle whose perimeter is 20 meters.

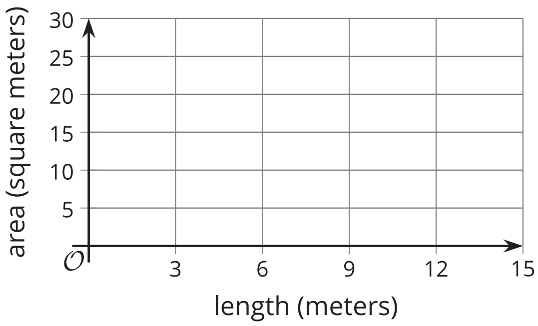
a. Complete the table. What do you notice about the areas?

length (meters)	width (meters)	area (square meters)
1	9	
3	7	
5		
7		
9		

b. Without calculating, predict whether the area of the rectangle will be greater or less than 25 square meters if the length is 5.25 meters.

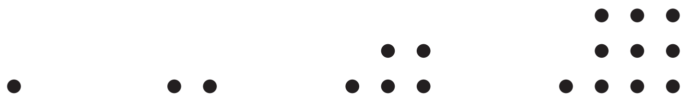
c. On the coordinate plane, plot the points for length and area from your table.

Do the values change in a linear way? Do they change in an exponential way?



(From Unit 6, Lesson 1.)

6. Here is a pattern of dots.



Step 0 Step 1 Step 2 Step 3

step	total number of dots
0	
1	
2	
3	

a. Complete the table.

b. How many dots will there be in Step 10?

c. How many dots will there be in Step n?

(From Unit 6, Lesson 2.)

7. Mai has a jar of quarters and dimes. She takes at least 10 coins out of the jar and has less than $2.00.

   a. Write a system of inequalities that represents the number of quarters, q, and the number of dimes, d, that Mai could have.

   b. Is it possible that Mai has each of the following combinations of coins? If so, explain or show how you know. If not, state which constraint—the amount of money or the number of coins—it does not meet.

      i. 3 quarters and 12 dimes

      ii. 4 quarters and 10 dimes

      iii. 2 quarters and 5 dimes

(From Unit 2, Lesson 25.)

8. A stadium can seat 63,026 people. For each game, the amount of money that the organization brings in through ticket sales is a function of the number of people, n, in attendance.

   If each ticket costs $30.00, find the domain and range of this function.

(From Unit 4, Lesson 10.)

Lesson 4: Comparing Quadratic and Exponential Functions

4.1: From Least to Greatest

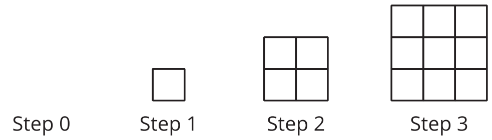
List these quantities in order, from least to greatest, without evaluating each expression. Be prepared to explain your reasoning.

A. 2^{10} B. 10^2 C. 2^9 D. 9^2

4.2: Which One Grows Faster?

- In Pattern A, the length and width of the rectangle grow by one small square from each step to the next.
- In Pattern B, the number of small squares doubles from each step to the next.
- In each pattern, the number of small squares is a function of the step number, n.

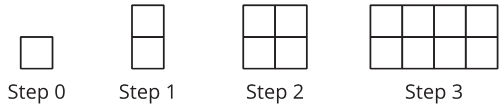
Pattern A



1. Write an equation to represent the number of small squares at Step n in Pattern A.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, step number	$f(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

Pattern B



1. Write an equation to represent the number of small squares at Step n in Pattern B.
2. Is the function linear, quadratic, or exponential?
3. Complete the table:

n, step number	$g(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

How would the two patterns compare if they continue to grow? Make 1–2 observations.

4.3: Comparing Two More Functions

Here are two functions: $p(x) = 6x^2$ and $q(x) = 3^x$.

Investigate the output of p and q for different values of x. For large enough values of x, one function will have a greater value than the other. Which function will have a greater value as x increases?

Support your answer with tables, graphs, or other representations.

Are you ready for more?

1. Jada says that some exponential functions grow more slowly than the quadratic function $f(x) = x^2$ as x increases. Do you agree with Jada? Explain your reasoning.

2. Could you have an exponential function $g(x) = b^x$ so that $g(x) < f(x)$ for all values of x?

Lesson 4 Summary

We have seen that the graphs of quadratic functions can curve upward. Graphs of exponential functions, with base larger than 1, also curve upward. To compare the two, let's look at the quadratic expression $3n^2$ and the exponential expression 2^n.

A table of values shows that $3n^2$ is initially greater than 2^n but 2^n eventually becomes greater.

n	$3n^2$	2^n
1	3	2
2	12	4
3	27	8
4	48	16
5	75	32
6	108	64
7	147	128
8	192	256

We also saw an explanation for why exponential growth eventually overtakes quadratic growth.

- When n increases by 1, the exponential expression 2^n always increases by a factor of 2.
- The quadratic expression $3n^2$ increases by different factors, depending on n, but these factors get smaller. For example, when n increases from 2 to 3, the factor is $\frac{27}{12}$ or 2.25. When n increases from 6 to 7, the factor is $\frac{147}{108}$ or about 1.36. As n increases to larger and larger values, $3n^2$ grows by a factor that gets closer and closer to 1.

A quantity that always doubles will eventually overtake a quantity growing by this smaller factor at each step.

Lesson 4: Comparing Quadratic and Exponential Functions

Cool Down: Comparing $5x^2$ and 2^x

Tyler completes the table comparing values of the expressions $5x^2$ and 2^x.

x	$5x^2$	2^x
1	5	2
2	20	4
3	45	8
4	80	16

Tyler concludes that $5x^2$ will always take larger values than 2^x for the same value of x. Do you agree? Explain or show your reasoning.

Unit 6 Lesson 4 Cumulative Practice Problems

1. The table shows values of the expressions $10x^2$ and 2^x.

x	$10x^2$	2^x
1	10	2
2	40	4
3	90	8
4	160	16
8		
10		
12		

   a. Describe how the values of each expression change as x increases.

   b. Predict which expression will have a greater value when:

      i. x is 8

      ii. x is 10

      iii. x is 12

   c. Find the value of each expression when x is 8, 10, and 12.

   d. Make an observation about how the values of the two expressions change as x becomes greater and greater.

2. Function f is defined by $f(x) = 1.5^x$. Function g is defined by $g(x) = 500x^2 + 345x$.

   a. Which function is quadratic? Which one is exponential?

   b. The values of which function will eventually be greater for larger and larger values of x?

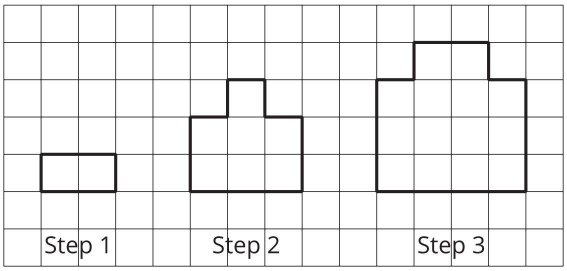
3. Create a table of values to show that the exponential expression $3(2)^x$ eventually overtakes the quadratic expression $3x^2 + 2x$.

4. The table shows the values of 4^x and $100x^2$ for some values of x.

Use the patterns in the table to explain why eventually the values of the exponential expression 4^x will overtake the values of the quadratic expression $100x^2$.

x	4^x	$100x^2$
1	4	100
2	16	400
3	64	900
4	256	1600
5	1024	2500

5. Here is a pattern of shapes. The area of each small square is 1 sq cm.



a. What is the area of the shape in Step 10?

b. What is the area of the shape in Step n?

c. Explain how you see the pattern growing.

(From Unit 6, Lesson 2.)

6. A bicycle costs \$240 and it loses $\frac{3}{5}$ of its value each year.

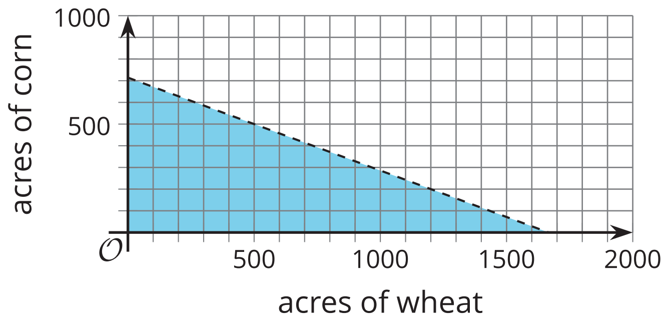
a. Write expressions for the value of the bicycle, in dollars, after 1, 2, and 3 years.

b. When will the bike be worth less than \$1?

c. Will the value of the bike ever be 0? Explain your reasoning.

(From Unit 5, Lesson 4.)

7. A farmer plants wheat and corn. It costs about \$150 per acre to plant wheat and about \$350 per acre to plant corn. The farmer plans to spend no more than \$250,000 planting wheat and corn. The total area of corn and wheat that the farmer plans to plant is less than 1200 acres.



This graph represents the inequality, $150w + 350c \leq 250{,}000$, which describes the cost constraint in this situation. Let w represent the number of acres of wheat and c represent the number of acres of corn.

a. The inequality, $x + y < 1{,}200$ represents the total area constraint in this situation. On the same coordinate plane, graph the solution to the inequality you wrote.

b. Use the graphs to find at least two possible combinations of the number of acres of wheat and the number of acres of corn that the farmer could plant.

c. The combination of 400 acres of wheat and 700 acres of corn meets one constraint in the situation but not the other constraint. Which constraint does this meet? Explain your reasoning.

(From Unit 2, Lesson 25.)

Lesson 5: Building Quadratic Functions to Describe Situations (Part 1)

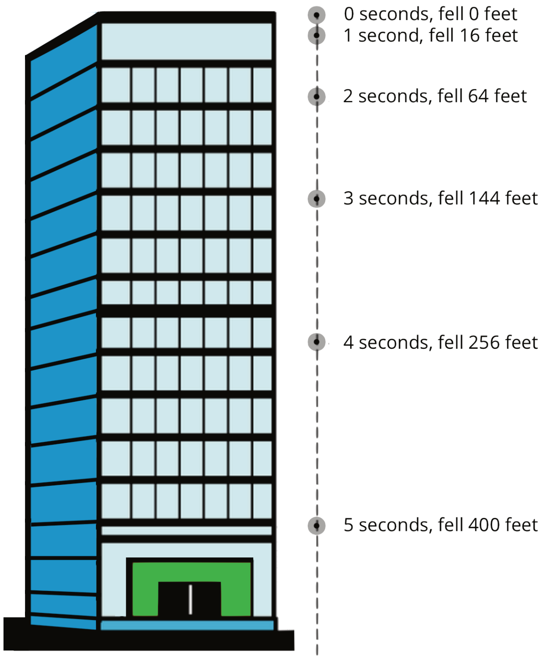
5.1: Notice and Wonder: An Interesting Numerical Pattern

Study the table. What do you notice? What do you wonder?

x	0	1	2	3	4	5
y	0	16	64	144	256	400

5.2: Falling from the Sky

A rock is dropped from the top floor of a 500-foot tall building. A camera captures the distance the rock traveled, in feet, after each second.



1. How far will the rock have fallen after 6 seconds? Show your reasoning.

2. Jada noticed that the distances fallen are all multiples of 16.
   She wrote down:

$$\begin{aligned} 16 &= 16 \cdot 1 \\ 64 &= 16 \cdot 4 \\ 144 &= 16 \cdot 9 \\ 256 &= 16 \cdot 16 \\ 400 &= 16 \cdot 25 \end{aligned}$$

   Then, she noticed that 1, 4, 9, 16, and 25 are $1^2, 2^2, 3^2, 4^2$ and 5^2.

   a. Use Jada's observations to predict the distance fallen after 7 seconds. (Assume the building is tall enough that an object dropped from the top of it will continue falling for at least 7 seconds.) Show your reasoning.

   b. Write an equation for the function, with d representing the distance dropped after t seconds.

5.3: Galileo and Gravity

Galileo Galilei, an Italian scientist, and other medieval scholars studied the motion of free-falling objects. The law they discovered can be expressed by the equation $d = 16 \cdot t^2$, which gives the distance fallen in feet, d, as a function of time, t, in seconds.

An object is dropped from a height of 576 feet.

1. How far does it fall in 0.5 seconds?

2. To find out where the object is after the first few seconds after it was dropped, Elena and Diego created different tables.

Elena's table:

time (seconds)	distance fallen (feet)
0	0
1	16
2	64
3	
4	
t	

Diego's table:

time (seconds)	distance from the ground (feet)
0	576
1	560
2	512
3	
4	
t	

a. How are the two tables are alike? How are they different?

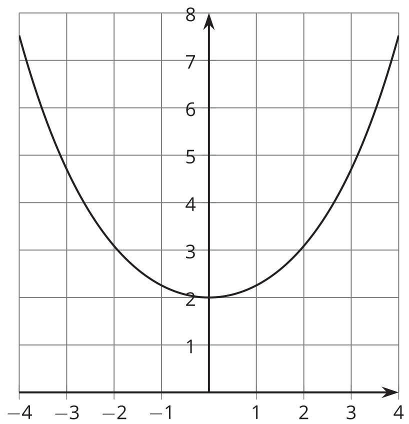
b. Complete Elena's and Diego's tables. Be prepared to explain your reasoning.

Are you ready for more?

Galileo correctly observed that gravity causes objects to fall in a way where the distance fallen is a quadratic function of the time elapsed. He got a little carried away, however, and assumed that a hanging rope or chain could also be modeled by a quadratic function.

Here is a graph of such a shape (called a catenary) along with a table of approximate values.

x	-4	-3	-2	-1	0	1	2	3	4
y	7.52	4.70	3.09	2.26	2	2.26	3.09	4.70	7.52



Show that an equation of the form $y = ax^2 + b$ cannot model this data well.

Lesson 5 Summary

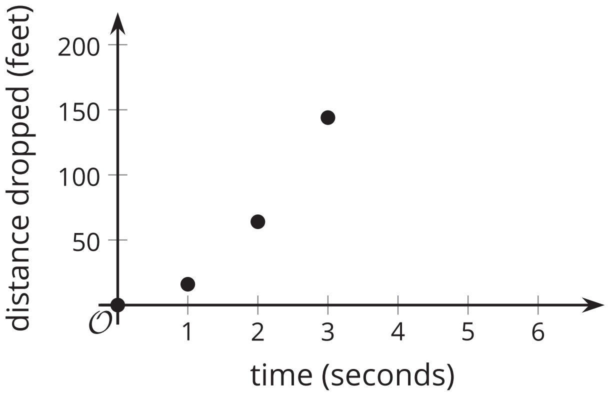
The distance traveled by a falling object in a given amount of time is an example of a quadratic function. Galileo is said to have dropped balls of different mass from the Leaning Tower of Pisa, which is about 190 feet tall, to show that they travel the same distance in the same time. In fact the equation $d = 16t^2$ models the distance d, in feet, that the cannonball falls after t seconds, no matter what its mass.

Because $16 \cdot 4^2 = 256$, and the tower is only 190 feet tall, the cannonball hits the ground before 4 seconds.

Here is a table showing how far the cannonball has fallen over the first few seconds.

time (seconds)	distance fallen (feet)
0	0
1	16
2	64
3	144

Here are the time and distance pairs plotted on a coordinate plane:



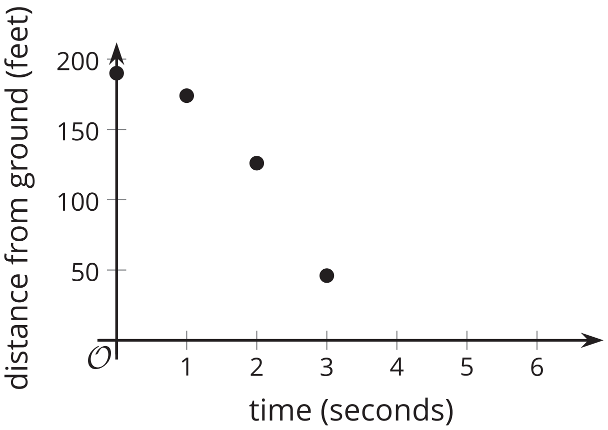
Notice that the distance fallen is increasing each second. The average rate of change is increasing each second, which means that the cannonball is speeding up over time. This comes from the influence of gravity, which is represented by the quadratic expression $16t^2$. It is the exponent 2 in that expression that makes it increase by larger and larger amounts.

Another way to study the change in the position of the cannonball is to look at its distance from the ground as a function of time.

Here is a table showing the distance from the ground in feet at 0, 1, 2, and 3 seconds.

time (seconds)	distance from the ground (feet)
0	190
1	174
2	126
3	46

Here are the time and distance pairs plotted on a graph:



The expression that defines the distance from the ground as a function of time is $190 - 16t^2$. It tells us that the cannonball's distance from the ground is 190 feet before it is dropped and has decreased by $16t^2$ when t seconds have passed.

Lesson 5: Building Quadratic Functions to Describe Situations (Part 1)

Cool Down: Where Will It Be?

The expression $16t^2$ represents the distance in feet an object falls after t seconds. The object is dropped from a height of 906 feet.

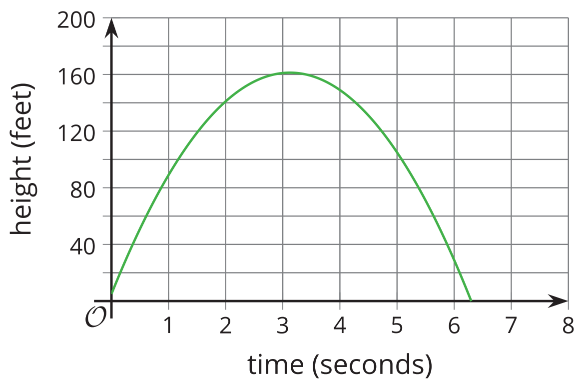
1. What is the height in feet of the object 2 seconds after it is dropped?

2. Write an expression representing the height of the object in feet t seconds after it is dropped.

Unit 6 Lesson 5 Cumulative Practice Problems

1. A rocket is launched in the air and its height, in feet, is modeled by the function h. Here is a graph representing h.

   Select **all** true statements about the situation.



   A. The rocket is launched from a height less than 20 feet above the ground.

   B. The rocket is launched from about 20 feet above the ground.

   C. The rocket reaches its maximum height after about 3 seconds.

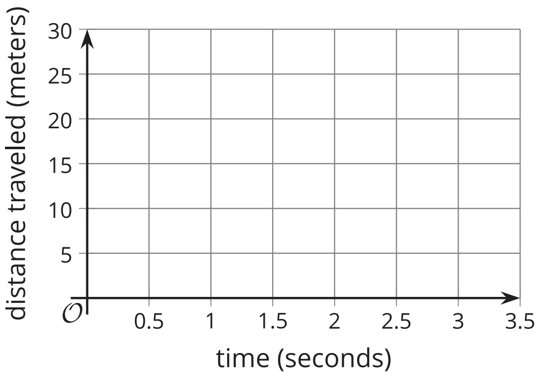
   D. The rocket reaches its maximum height after about 160 seconds.

   E. The maximum height of the rocket is about 160 feet.

2. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

   a. Complete the table and plot the data on the coordinate plane.

t (seconds)	d (meters)
0	
0.5	
1	
1.5	
2	



   b. Is the baseball traveling at a constant speed? Explain how you know.

3. A rock is dropped from a bridge over a river. Which table could represent the distance in feet fallen as a function of time in seconds?

Table A

time (seconds)	distance fallen (feet)
1	0
2	48
3	96
4	144

Table B

time (seconds)	distance fallen (feet)
1	0
2	16
3	64
4	144

Table C

time (seconds)	distance fallen (feet)
1	180
2	132
3	84
4	36

Table D

time (seconds)	distance fallen (feet)
1	180
2	164
3	116
4	36

A. Table A

B. Table B

C. Table C

D. Table D

4. Determine whether $5n^2$ or 3^n will have the greater value when:

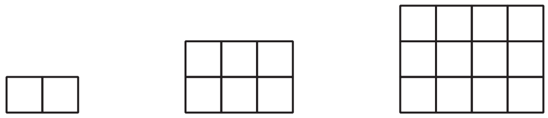
a. $n = 1$

b. $n = 3$

c. $n = 5$

(From Unit 6, Lesson 4.)

5. Select **all** of the expressions that give the number of small squares in Step n.



Step 1 Step 2 Step 3

A. $2n$

B. n^2

C. $n+1$

D. n^2+1

E. $n(n+1)$

F. n^2+n

G. $n+n+1$

(From Unit 6, Lesson 3.)

6. A small ball is dropped from a tall building. Which equation could represent the ball's height, h, in feet, relative to the ground, as a function of time, t, in seconds?

A. $h=100-16t$

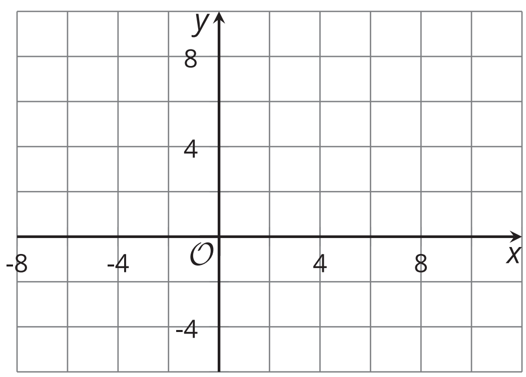
B. $h=100-16t^2$

C. $h=100-16^t$

D. $h=100-\frac{16}{t}$

7. Use the rule for function f to draw its graph.

$$f(x)=\begin{cases} 2, & -5 \le x < -2 \\ 6, & -2 \le x < 4 \\ x, & 4 \le x < 8 \end{cases}$$



(From Unit 4, Lesson 12.)

8. Diego claimed that $10 + x^2$ is always greater than 2^x and used this table as evidence.

   Do you agree with Diego?

x	$10 + x^2$	2^x
1	11	2
2	14	4
3	19	8
4	26	16

(From Unit 6, Lesson 4.)

9. The table shows the height, in centimeters, of the water in a swimming pool at different times since the pool started to be filled.

   a. Does the height of the water increase by the same amount each minute? Explain how you know.

   b. Does the height of the water increase by the same factor each minute? Explain how you know.

minutes	height
0	150
1	150.5
2	151
3	151.5

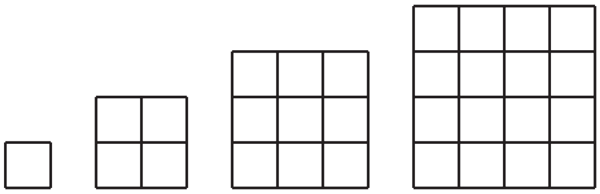
(From Unit 5, Lesson 2.)

Family Support Materials

Introduction to Quadratic Functions

In this unit, students learn about quadratic functions. Earlier, they learned about linear functions that grow by repeatedly adding or subtracting the same amount and exponential functions that grow by repeatedly multiplying by the same amount.

Quadratic functions also change in a predictable way. Here, the number of small squares in each step is increasing by 3, then 5, then 7, and so on. How many squares are in Step 10? How many in Step n?

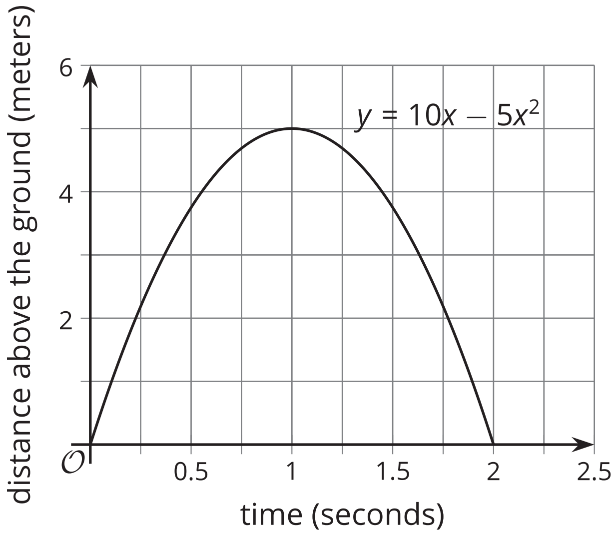


Step 1 Step 2 Step 3 Step 4

Here is a table that shows the pattern.

step number	1	2	3	4	10	n
number of small squares	1	4	9	4×4 or 16	10×10 or 100	$n \times n$ or n^2

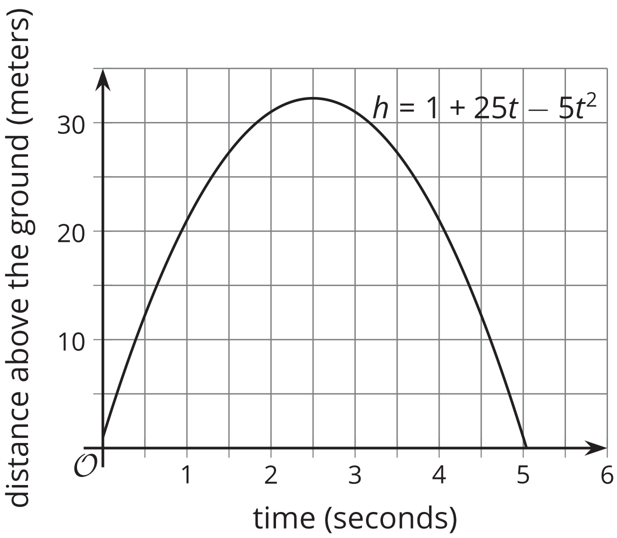
In this unit, students will also learn about some real-world situations that can be modeled by quadratic functions. For example, when you toss a ball up in the air, its distance above the ground as time passes can be modeled by a quadratic function. Study the graph. The ball starts on the ground because the height is 0 when time is 0. The ball lands back on the ground after 2 seconds. After 1 second, the ball is 5 meters in the air.



Both of the following expressions give the ball's distance above the ground: $5x(2 - x)$ and $10x - 5x^2$, where x represents the number of seconds since it was thrown. Quadratic expressions are most recognizable when you can see the "squared term," $-5x^2$, as shown in $10x - 5x^2$.

Your student will learn more about quadratics in the next unit.

Here is a task to try with your student:
The equation $h = 1 + 25t^2 - 5t^2$ models the height in meters of a model rocket t seconds after it is launched in the air. Here is a graph representing the equation.



1. What was the height of the rocket above the ground at the time it was launched?

2. How high did it go up in the air?

3. When did the rocket land back on the ground?

Solution:

1. 1 meter

2. about 32 meters

3. a little more than 5 seconds after launch