Illustrative Mathematics

Algebra 1
Introduction to Quadratics

Lessons 6-10

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Illustrative Mathematics

Lesson 6: Building Quadratic Functions to Describe Situations (Part 2)

6.1: Sky Bound

A cannon is 10 feet off the ground. It launches a cannonball straight up with a velocity of 406 feet per second.

Imagine that there is no gravity and that the cannonball continues to travel upward with the same velocity.

1. Complete the table with the heights of the cannonball at different times.

seconds	0	1	2	3	4	5	t
distance above ground (feet)	10						

2. Write an equation to model the distance in feet, d, of the ball t seconds after it was fired from the cannon if there was no gravity.

6.2: Tracking a Cannonball

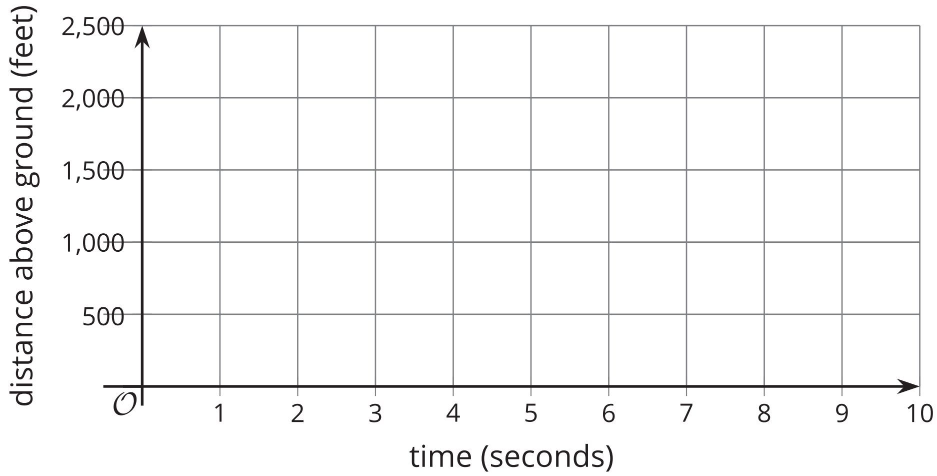
Earlier, you completed a table that represents the height of a cannonball, in feet, as a function of time, in seconds, if there was no gravity.

1. This table shows the actual heights of the ball at different times.

seconds	0	1	2	3	4	5
distance above ground (feet)	10	400	758	1,084	1,378	1,640

   Compare the values in this table with those in the table you completed earlier. Make at least 2 observations.

2. a. Plot the two sets of data you have on the same coordinate plane.



b. How are the two graphs alike? How are they different?

3. Write an equation to model the actual distance d, in feet, of the ball t seconds after it was fired from the cannon. If you get stuck, consider the differences in distances and the effects of gravity from a previous lesson.

6.3: Graphing Another Cannonball

The function defined by $d = 50 + 312t - 16t^2$ gives the height in feet of a cannonball t seconds after the ball leaves the cannon.

1. What do the terms 50, $312t$, and -$16t^2$ tell us about the cannonball?

2. Use graphing technology to graph the function. Adjust the graphing window to the following boundaries: $0 < x < 25$ and $0 < y < 2{,}000$.

3. Observe the graph and:

    a. Describe the shape of the graph. What does it tell us about the movement of the cannonball?

    b. Estimate the maximum height the ball reaches. When does this happen?

    c. Estimate when the ball hits the ground.

4. What domain is appropriate for this function? Explain your reasoning.

Are you ready for more?

If the cannonball were fired at 800 feet per second, would it reach a mile in height? Explain your reasoning.

Lesson 6 Summary

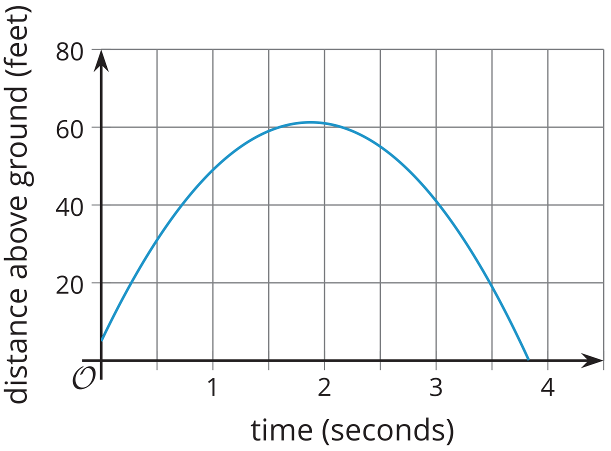
In this lesson, we looked at the height of objects that are launched upward and then come back down because of gravity.

An object is thrown upward from a height of 5 feet with a velocity of 60 feet per second. Its height $h(t)$ in feet after t seconds is modeled by the function $h(t) = 5 + 60t - 16t^2$.

- The linear expression $5 + 60t$ represents the height the object would have at time t if there were no gravity. The object would keep going up at the same speed at which it was thrown. The graph would be a line with a slope of 60 which relates to the constant speed of 60 feet per second.
- The expression $-16t^2$ represents the effect of gravity, which eventually causes the object to slow down, stop, and start falling back again.

Notice the graph intersects the vertical axis at 5, which means the object was thrown into the air from 5 feet off the ground. The graph indicates that the object reaches its peak height of about 60 feet after a little less than 2 seconds. That peak is the point on the graph where the function reaches a maximum value. At that point, the curve changes direction, and the output of the function changes from increasing to decreasing. We call that point the **vertex** of the graph.

Here is the graph of h.



The graph representing any quadratic function is a special kind of "U" shape called a *parabola*. You will learn more about the geometry of parabolas in a future course. Every parabola has a vertex, because there is a point where it changes direction—from increasing to decreasing, or the other way around.

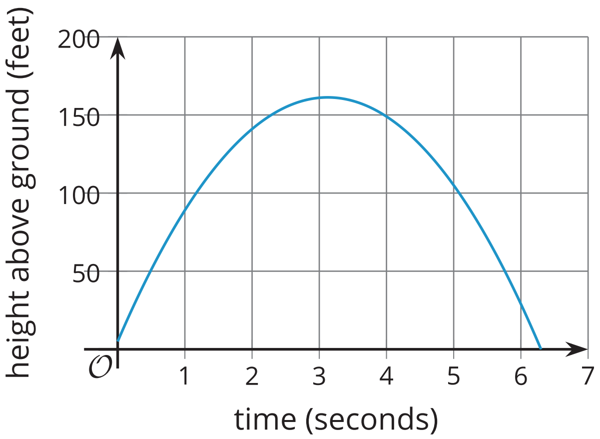
The object hits the ground a little before 4 seconds. That time corresponds to the horizontal intercept of the graph. An input value that produces an output of 0 is called a **zero** of the function. A zero of the function h is approximately 3.8, because $h(3.8) \approx 0$.

In this situation, input values less than 0 seconds or more than about 3.8 seconds would not be meaningful, so an appropriate domain for this function would include all values of t between 0 and about 3.8.

Lesson 6: Building Quadratic Functions to Describe Situations (Part 2)

Cool Down: Rocket in the Air

The height, h, of a stomp rocket (propelled by a short blast of air) above the ground after t seconds is given by the equation $h(t) = 5 + 100t - 16t^2$. Here is a graph that represents h.



1. How does the 5 in the equation relate to the graph?

2. What does $100t$ in the equation mean in terms of the rocket?

3. What does the $-16t^2$ mean in terms of the rocket?

4. About when does the rocket hit the ground?

Unit 6 Lesson 6 Cumulative Practice Problems

1. The height of a diver above the water, is given by $h(t) = -5t^2 + 10t + 3$, where t is time measured in seconds and $h(t)$ is measured in meters. Select **all** statements that are true about the situation.

   A. The diver begins 5 meters above the water.

   B. The diver begins 3 meters above the water.

   C. The function has 1 zero that makes sense in this situation.

   D. The function has 2 zeros that make sense in this situation.

   E. The graph that represents h starts at the origin and curves upward.

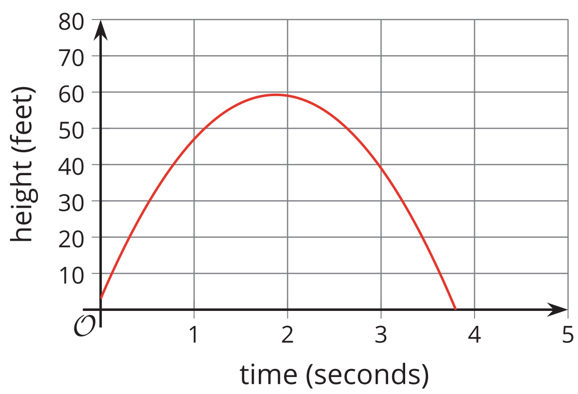
   F. The diver begins at the same height as the water level.

2. The height of a baseball, in feet, is modeled by the function h given by the equation $h(t) = 3 + 60t - 16t^2$. The graph of the function is shown.

   a. About when does the baseball reach its maximum height?

   b. About how high is the maximum height of the baseball?

   c. About when does the ball hit the ground?



3. *Technology required.* Two rocks are launched straight up in the air. The height of Rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$. The height of Rock B is given by g, where $g(t) = 5 + 20t - 16t^2$. In both functions, t is time measured in seconds and height is measured in feet.

   Use graphing technology to graph both equations. Determine which rock hits the ground first and explain how you know.

4. Each expression represents an object's distance from the ground in meters as a function of time, t, in seconds.

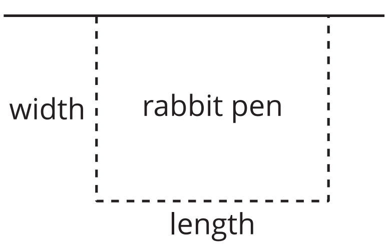
   Object A: $-5t^2 + 25t + 50$

   Object B: $-5t^2 + 50t + 25$

   a. Which object was launched with the greatest vertical speed?

   b. Which object was launched from the greatest height?

5. Tyler is building a pen for his rabbit on the side of the garage. He needs to fence in three sides and wants to use 24 ft of fencing.



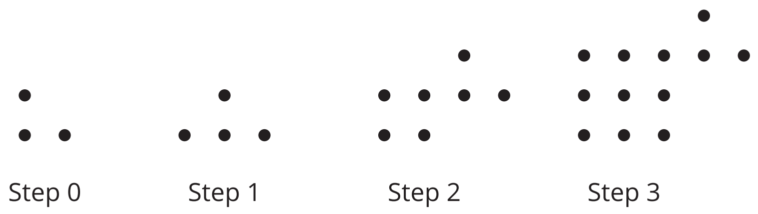
   a. The table show some possible lengths and widths. Complete each area.

   b. Which length and width combination should Tyler choose to give his rabbit the most room?

length (ft)	width (ft)	area (sq ft)
8	8	
10	7	
12	6	
14	5	
16	4	

(From Unit 6, Lesson 1.)

6. Here is a pattern of dots.



step	total number of dots
0	
1	
2	
3	

a. Complete the table.

b. How many dots will there be in Step 10?

c. How many dots will there be in Step n?

(From Unit 6, Lesson 2.)

7. The function f is defined by $f(x) = 2^x$ and the function g is defined by $g(x) = x^2 + 16$.

a. Find the values of f and g when x is 4, 5, and 6.

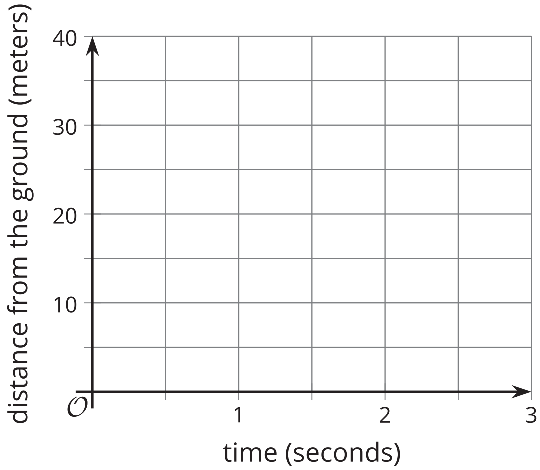
b. Will the values of f always be greater than the values of g? Explain how you know.

(From Unit 6, Lesson 4.)

8. Han accidentally drops his water bottle from the balcony of his apartment building. The equation $d = 32 - 5t^2$ gives the distance from the ground, d, in meters after t seconds.

   a. Complete the table and plot the data on the coordinate plane.

t (seconds)	d (meters)
0	
0.5	
1	
1.5	
2	



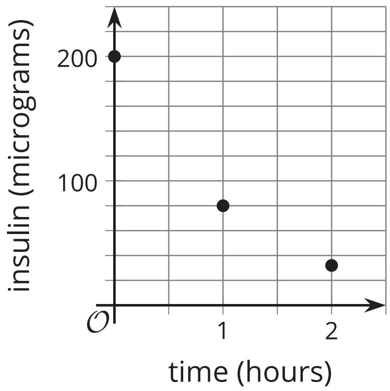
   b. Is the water bottle falling at a constant speed? Explain how you know.

(From Unit 6, Lesson 5.)

9. The graph shows how much insulin, in micrograms (mcg), is in a patient's body after receiving an injection.

   a. Write an equation giving the number of mcg of insulin, m, in the patient's body h hours after receiving the injection.

   b. After 3 hours, will the patient still have at least 10 mcg of insulin in their body? Explain how you know.



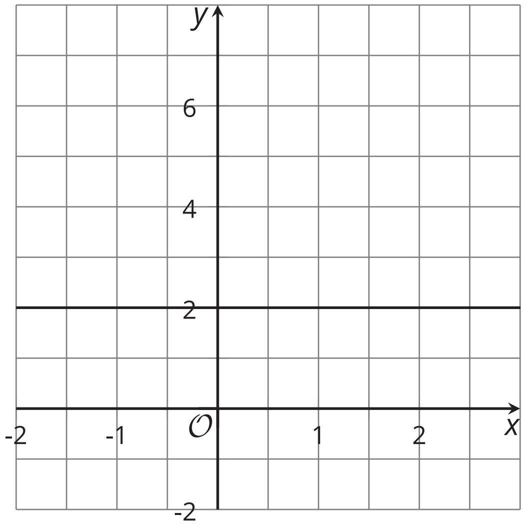
(From Unit 5, Lesson 6.)

Lesson 7: Building Quadratic Functions to Describe Situations (Part 3)

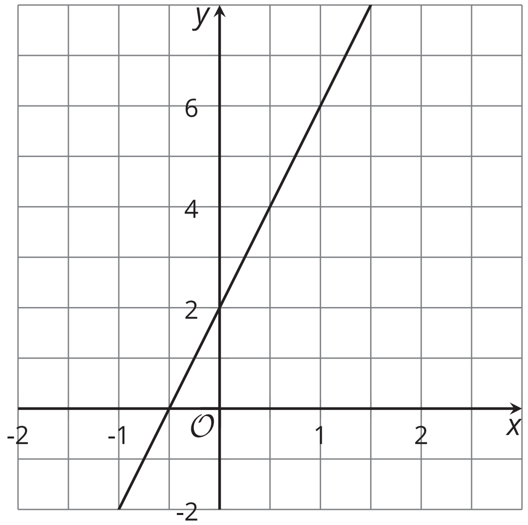
7.1: Which One Doesn't Belong: Graphs of Four Functions

Which one doesn't belong?

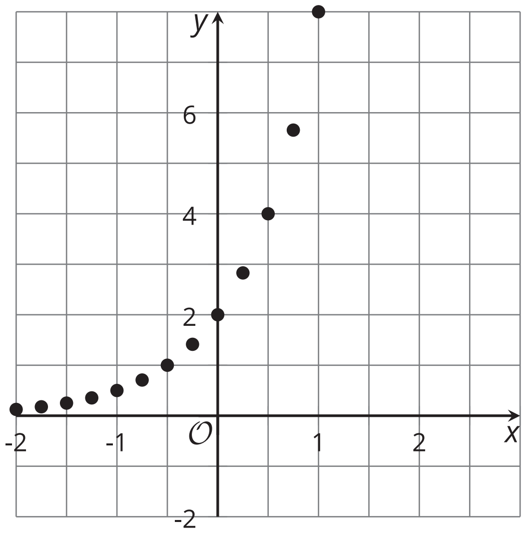
A



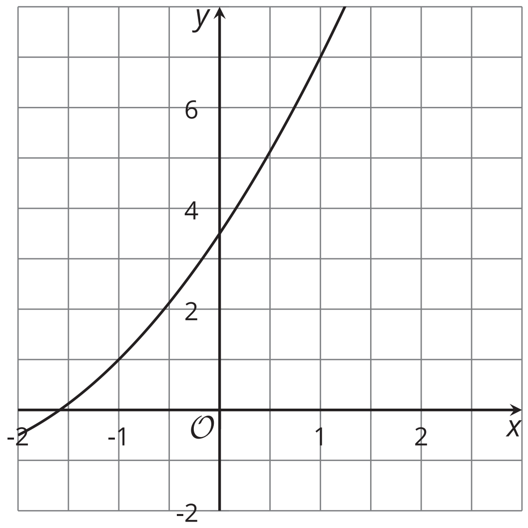
B



C



D



7.2: What Price to Charge?

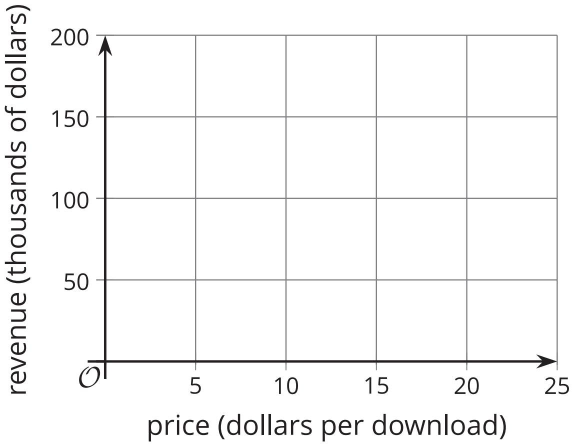
A company that sells movies online is deciding how much to charge customers to download a new movie. Based on data from previous sales, the company predicts that if they charge x dollars for each download, then the number of downloads, in thousands, is $18 - x$.

1. Complete the table to show the predicted number of downloads at each listed price. Then, find the revenue at each price. The first row has been completed for you.

price (dollars per download)	number of downloads (thousands)	revenue (thousands of dollars)
3	15	45
5		
10		
12		
15		
18		
x		

2. Is the relationship between the price of the movie and the revenue (in thousands of dollars) quadratic? Explain how you know.

3. Plot the points that represent the revenue, r, as a function of the price of one download in dollars, x.



4. What price would you recommend the company charge for a new movie? Explain your reasoning.

Are you ready for more?

The function that uses the price (in dollars per download) x to determine the number of downloads (in thousands) $18 - x$ is an example of a demand function and its graph is known. Economists are interested in factors that can affect the demand function and therefore the price suppliers wish to set.

1. What are some things that could increase the number of downloads predicted for the same given prices?

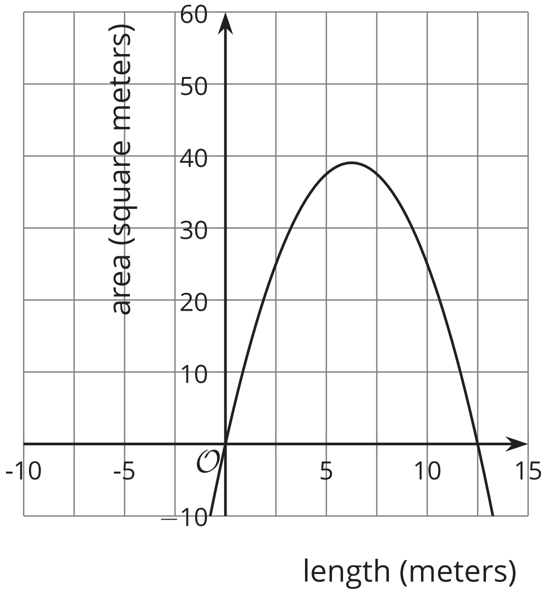
2. If the demand shifted so that we predicted $20 - x$ thousand downloads at a price of x dollars per download, what do you think will happen to the price that gives the maximum revenue? Check what actually happens.

7.3: Domain, Vertex, and Zeros

Here are 4 sets of descriptions and equations that represent some familiar quadratic functions. The graphs show what a graphing technology may produce when the equations are graphed. For each function:

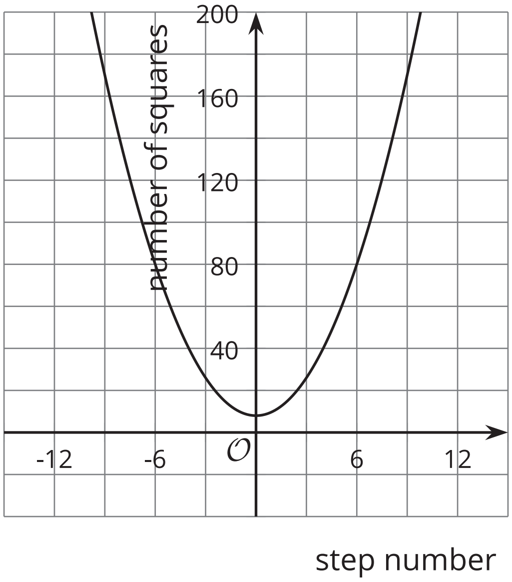
- Describe a domain that is appropriate for the situation. Think about any upper or lower limits for the input, as well as whether all numbers make sense as the input. Then, describe how the graph should be modified to show the domain that makes sense.
- Identify or estimate the vertex on the graph. Describe what it means in the situation.
- Identify or estimate the zeros of the function. Describe what it means in the situation.

1. The area of rectangle with a perimeter of 25 meters and a side length x:
$A(x) = x \cdot \frac{(25-2x)}{2}$



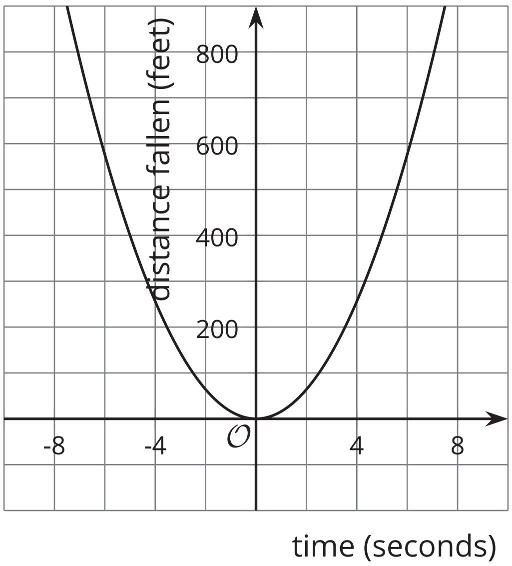
- Domain:
- Vertex:
- Zeros:

2. The number of squares as a function of step number n: $f(n) = n^2 + 4$



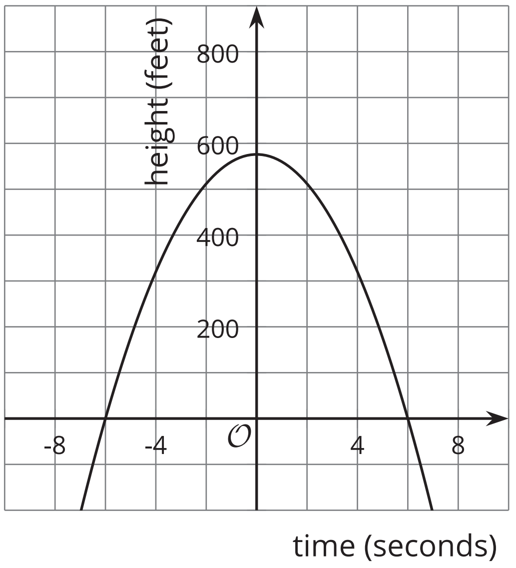
- Domain:
- Vertex:
- Zeros:

3. The distance in feet that an object has fallen t seconds after being dropped: $g(t) = 16t^2$



- Domain:
- Vertex:
- Zeros:

4. The height in feet of an object t seconds after being dropped: $h(t) = 576 - 16t^2$



- Domain:

- Vertex:

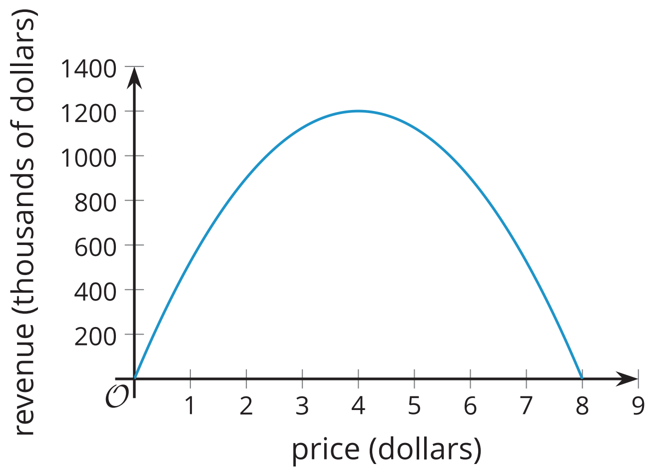
- Zeros:

Lesson 7 Summary

Quadratic functions often come up when studying revenue. (*Revenue* means the money collected when someone sells something.)

Suppose we are selling raffle tickets and deciding how much to charge for each ticket. When the price of the tickets is higher, typically fewer tickets will be sold.

Let's say that with a price of d dollars, it is possible to sell $600 - 75d$ tickets. We can find the revenue by multiplying the price by the number of tickets expected to be sold. A function that models the revenue r collected is $r(d) = d(600 - 75d)$. Here is a graph that represents the function.



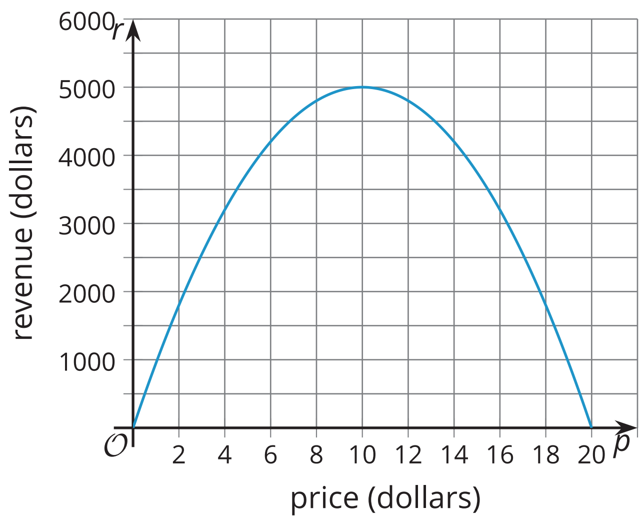
It makes sense that the revenue goes down after a certain point, since if the price is too high nobody will buy a ticket. From the graph, we can tell that the greatest revenue, \$1,200, comes from selling the tickets for \$4 each.

We can also see that the domain of the function r is between 0 and 8. This makes sense because the cost of the tickets can't be negative, and if the price were more than \$8, the model does not work, as the revenue collected cannot be negative. (A negative revenue would mean the number of tickets sold is negative, which is not possible.)

Lesson 7: Building Quadratic Functions to Describe Situations (Part 3)

Cool Down: Making the Greatest Revenue

This graph represents the revenue in dollars that a company expects if they sell their product for p dollars.



1. Based on this model, which price would generate more revenue for the company, \$5 or \$17? Explain how you know.

2. At what price should the company sell their product if they wish to make as much revenue as possible? How much revenue will they make?

3. What is an appropriate domain for the function? Explain how you know.

Unit 6 Lesson 7 Cumulative Practice Problems

1. Based on past musical productions, a theater predicts selling $400 - 8p$ tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many tickets the theater expects to sell and what revenues it expects to receive at the given ticket prices.

ticket price (dollars)	number of tickets sold	revenue (dollars)
5		
10		
15		
20		
30		
45		
50		
p		

   b. For which ticket prices will the theater earn no revenue? Explain how you know.

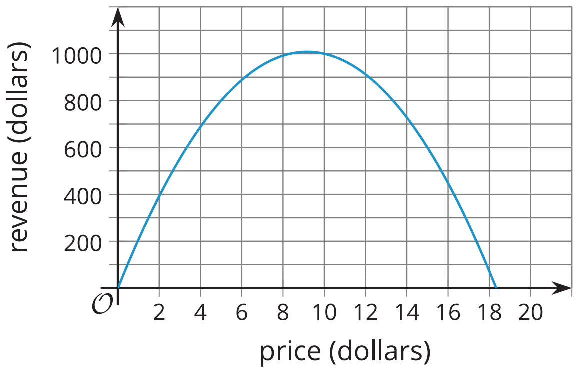
   c. At what ticket prices should the theater sell the tickets if it must earn at least $3,200 in revenue to break even (to not lose money) on the musical production? Explain how you know.

2. A company sells running shoes. If the price of a pair of shoes in dollars is p, the company estimates that it will sell $50{,}000 - 400p$ pairs of shoes.

   Write an expression that represents the revenue in dollars from selling running shoes if a pair of shoes is priced at p dollars.

3. The function f represents the revenue in dollars the school can expect to receive if it sells $220 - 12x$ coffee mugs for x dollars each.

   Here is the graph of f.



   Select **all** the statements that describe this situation.

   A. At \$2 per coffee mug, the revenue will be \$196.

   B. The school expects to sell 160 mugs if the price is \$5 each.

   C. The school will lose money if it sells the mugs for more than \$10 each.

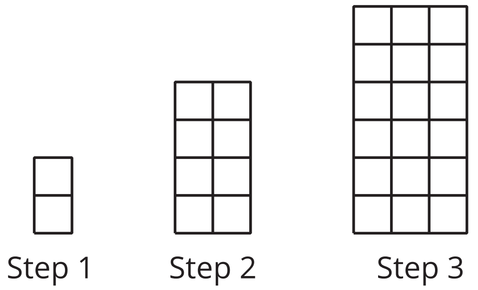
   D. The school will earn about \$1,000 if it sells the mugs for \$10 each.

   E. The revenue will be more than \$700 if the price is between \$4 and \$14.

   F. The expected revenue will increase if the price per mug is greater than \$10.

4. a. Write an equation to represent the relationship between the step number, n, and the number of small squares, y.

   Briefly describe how each part of the equation relates to the pattern.



   b. Is the relationship between the step number and number of small squares quadratic? Explain how you know.

(From Unit 6, Lesson 3.)

5. *Technology required.* A small marshmallow is launched straight up in the air with a slingshot. The function h, given by the equation $h(t) = 5 + 20t - 5t^2$, describes the height of the marshmallow in meters as a function of time, t, in seconds since it was launched.

    a. Use graphing technology to graph the function h.

    b. About when does the marshmallow reach its maximum height?

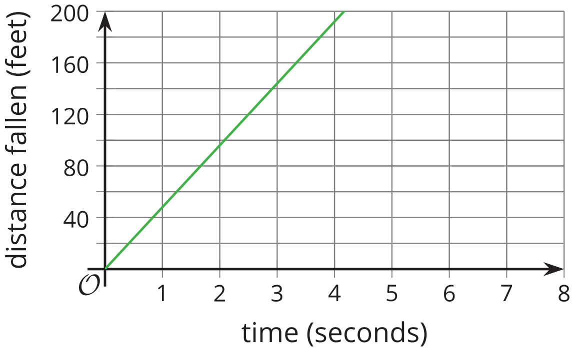
    c. About how long does it take before the marshmallow hits the ground?

    d. What domain makes sense for the function h in this situation?

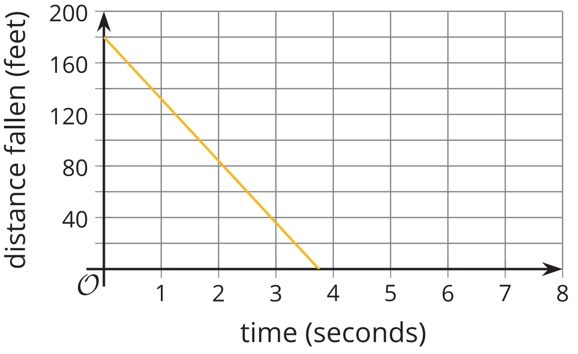
(From Unit 6, Lesson 5.)

6. A rock is dropped from a bridge over a river. Which graph could represent the distance fallen, in feet, as a function of time in seconds?

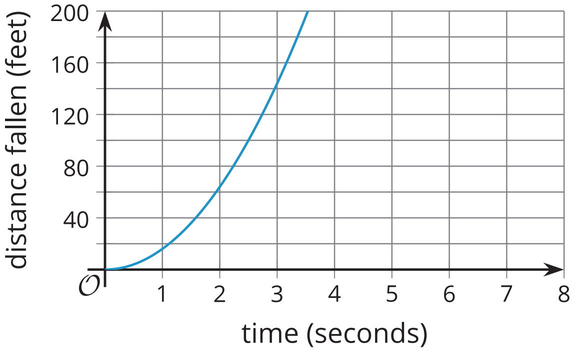
Graph A



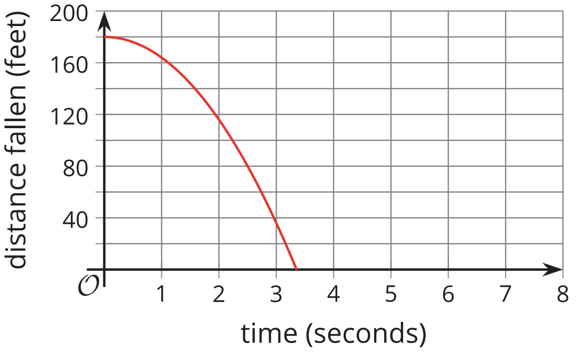
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

(From Unit 6, Lesson 5.)

7. A bacteria population, p, is modeled by the equation $p = 100{,}000 \cdot 2^d$, where d is the number of days since the population was first measured.

Select **all** statements that are true in this situation.

A. $100{,}000 \cdot 2^{-2}$ represents the bacteria population 2 days before it was first measured.

B. The bacteria population 3 days before it was first measured was 800,000.

C. The population was more than 1,000 one week before it was first measured.

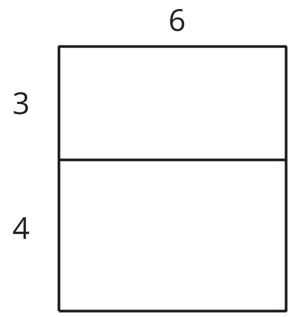
D. The population was more than 1,000,000 one week after it was first measured.

E. The bacteria population 4 days before it was first measured was 6,250.

(From Unit 5, Lesson 7.)

Lesson 8: Equivalent Quadratic Expressions

8.1: Diagrams of Products

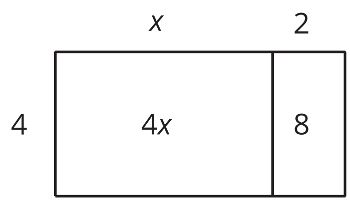


1. Explain why the diagram shows that $6(3+4) = 6 \cdot 3 + 6 \cdot 4$.

2. Draw a diagram to show that $5(x+2) = 5x + 10$.

8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.



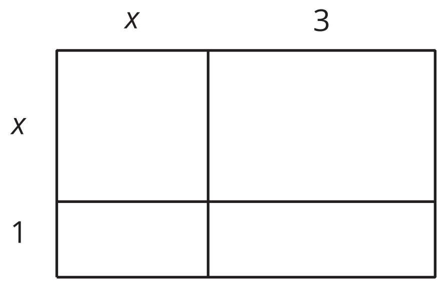
1. Draw a diagram to show that $n(2n+5)$ and $2n^2+5n$ are equivalent expressions.

2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

   a. $6\left(\frac{1}{3}n+2\right)$ b. $p(4p+9)$ c. $5r\left(r+\frac{3}{5}\right)$ d. $(0.5w+7)w$

8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x + 1$ and $x + 3$. Use this diagram to show that $(x + 1)(x + 3)$ and $x^2 + 4x + 3$ are equivalent expressions.



2. Draw diagrams to help you write an equivalent expression for each of the following:
   a. $(x + 5)^2$

   b. $2x(x + 4)$

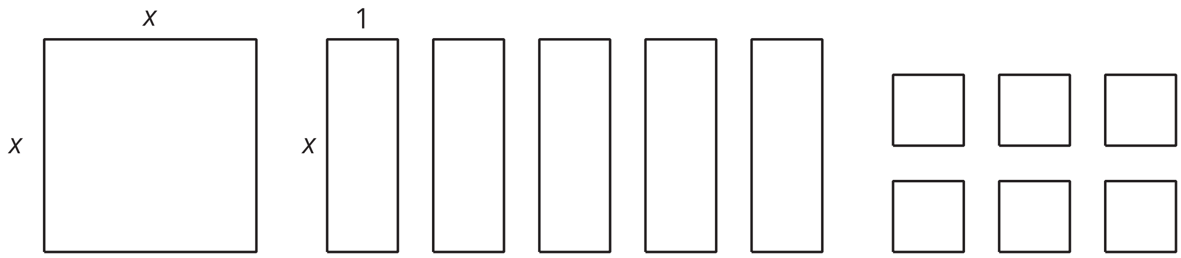
   c. $(2x + 1)(x + 3)$

   d. $(x + m)(x + n)$

3. Write an equivalent expression for each expression without drawing a diagram:
   a. $(x + 2)(x + 6)$

   b. $(x + 5)(2x + 10)$

Are you ready for more?



1. Is it possible to arrange an x by x square, five x by 1 rectangles and six 1 by 1 squares into a single large rectangle? Explain or show your reasoning.

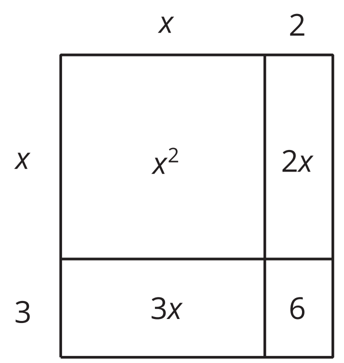
2. What does this tell you about an equivalent expression for $x^2 + 5x + 6$?

3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at x dollars can be expressed with $x(18-x)$, which can also be written as $18x - x^2$. The former is a product of x and $18-x$, and the latter is a difference of $18x$ and x^2, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x+2)(x+3)$. We can write an equivalent expression by thinking about each factor, the $(x+2)$ and $(x+3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x+2)$ and $(x+3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x+2)(x+3)$ is equivalent to $x^2+2x+3x+6$, or to x^2+5x+6.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the x and the 2 in $x+2$) is multiplied by every term in the other factor (the x and the 3 in $x+3$).

$$\begin{aligned}
&(x+2)(x+3)\\
&= x(x+3)+2(x+3)\\
&= x^2+3x+2x+(2)(3)\\
&= x^2+(3+2)x+(2)(3)
\end{aligned}$$

In general, when a quadratic expression is written in the form of $(x+p)(x+q)$, we can apply the distributive property to rewrite it as $x^2+px+qx+pq$ or $x^2+(p+q)x+pq$.

Lesson 8: Equivalent Quadratic Expressions

Cool Down: Writing Equivalent Expressions

1. Use a diagram to show that $(3x + 1)(x + 2)$ is equivalent to $3x^2 + 7x + 2$.

2. Is $(x + 4)^2$ equivalent to $2x^2 + 8x + 8$? Explain or show your reasoning.

Unit 6 Lesson 8 Cumulative Practice Problems

1. Draw a diagram to show that $(2x+5)(x+3)$ is equivalent to $2x^2+11x+15$.

2. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

   A. $(x+2)(x+6)$
   B. $(2x+8)(x+2)$
   C. $(x+8)(x+4)$
   D. $(x+2)(2x+6)$

   1. $x^2+12x+32$
   2. $2x^2+10x+12$
   3. $2x^2+12x+16$
   4. $x^2+8x+12$

3. Select **all** expressions that are equivalent to x^2+4x.

   A. $x(x+4)$
   B. $(x+2)^2$
   C. $(x+x)(x+4)$
   D. $(x+2)^2-4$
   E. $(x+4)x$

4. Tyler drew a diagram to expand $(x+5)(2x+3)$.

   a. Explain Tyler's mistake.

   b. What is the correct expanded form of $(x+5)(2x+3)$?

	$2x$	3
x	$2x^2$	$3x$
5	$7x$	8

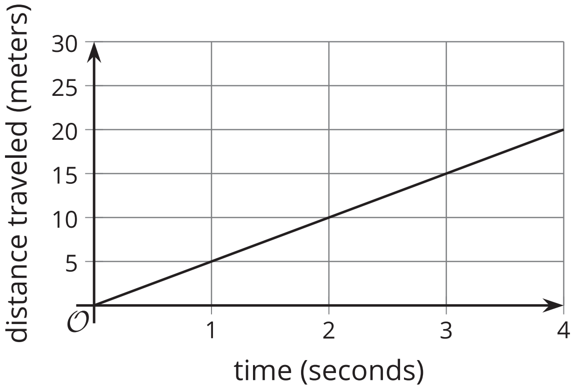
5. Explain why the values of the exponential expression 3^x will eventually overtake the values of the quadratic expression $10x^2$.

(From Unit 6, Lesson 4.)

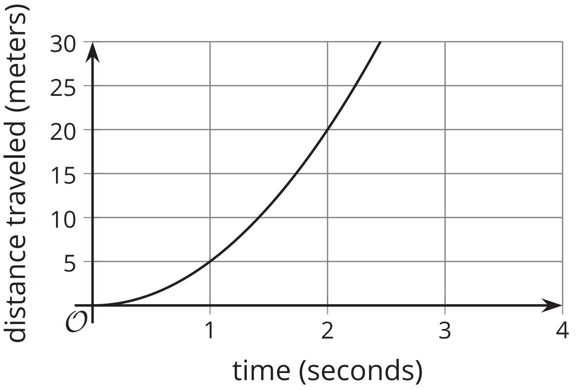
6. A baseball travels d meters t seconds after being dropped from the top of a building. The distance traveled by the baseball can be modeled by the equation $d = 5t^2$.

Which graph could represent this situation? Explain how you know.

Graph A



Graph B



(From Unit 6, Lesson 5.)

7. Consider a function q defined by $q(x) = x^2$. Explain why negative values are not included in the range of q.

(From Unit 4, Lesson 10.)

8. Based on past concerts, a band predicts selling $600 - 10p$ concert tickets when each ticket is sold at p dollars.

   a. Complete the table to find out how many concert tickets the band expects to sell and what revenues it expects to receive at the given ticket prices.

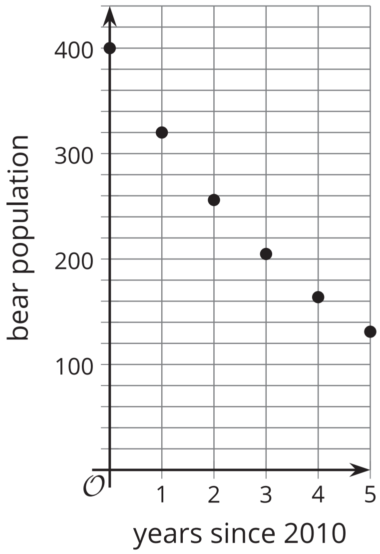
ticket price (dollars)	number of tickets	revenue (dollars)
10		
15		
20		
30		
35		
45		
50		
60		
p		

   b. In this model, at what ticket prices will the band earn no revenue at all?

   c. At what ticket prices should the band sell the tickets if it must earn at least 8,000 dollars in revenue to break even (to not lose money) on a given concert. Explain how you know.

(From Unit 6, Lesson 7.)

9. A population of bears decreases exponentially.



   a. What is the annual factor of decrease for the bear population? Explain how you know.

   b. Using function notation, represent the relationship between the bear population, b, and the number of years since the population was first measured, t. That is, find a function, f, so that $b = f(t)$.

(From Unit 5, Lesson 8.)

10. Equations defining functions a, b, c, d, and f are shown here.

Select **all** the equations that represent exponential functions.

A. $a(x) = 2^3 \cdot x$

B. $b(t) = \left(\frac{2}{3}\right)^t$

C. $c(m) = \frac{1}{5} \cdot 2^m$

D. $d(x) = 3x^2$

E. $f(t) = 3 \cdot 2^t$

(From Unit 5, Lesson 8.)

Lesson 9: Standard Form and Factored Form

9.1: Math Talk: Opposites Attract

Solve each equation mentally.

$40 - 8 = 40 + n$

$25 + \text{-}100 = 25 - n$

$3 - \frac{1}{2} = 3 + n$

$72 - n = 72 + 6$

9.2: Finding Products of Differences

1. Show that $(x-1)(x-1)$ and $x^2 - 2x + 1$ are equivalent expressions by drawing a diagram or applying the distributive property. Show your reasoning.

2. For each expression, write an equivalent expression. Show your reasoning.
    a. $(x+1)(x-1)$

    b. $(x-2)(x+3)$

    c. $(x-2)^2$

9.3: What is the Standard Form? What is the Factored Form?

The quadratic expression $x^2 + 4x + 3$ is written in **standard form**.

Here are some other quadratic expressions. The expressions on the left are written in standard form and the expressions on the right are not.

Written in standard form:	Not written in standard form:
$x^2 - 1$	$(2x + 3)x$
$x^2 + 9x$	$(x + 1)(x - 1)$
$\frac{1}{2}x^2$	$3(x - 2)^2 + 1$
$4x^2 - 2x + 5$	$-4(x^2 + x) + 7$
$-3x^2 - x + 6$	$(x + 8)(-x + 5)$
$1 - x^2$	

1. What are some characteristics of expressions in standard form?

2. $(x + 1)(x - 1)$ and $(2x + 3)x$ in the right column are quadratic expressions written in **factored form**. Why do you think that form is called factored form?

Are you ready for more?

Which quadratic expression can be described as being both standard form and factored form? Explain how you know.

Lesson 9 Summary

A quadratic function can often be represented by many equivalent expressions. For example, a quadratic function f might be defined by $f(x) = x^2 + 3x + 2$. The quadratic expression $x^2 + 3x + 2$ is called the **standard form**, the sum of a multiple of x^2 and a linear expression ($3x + 2$ in this case).

In general, standard form is

$$ax^2 + bx + c$$

We refer to a as the coefficient of the squared term x^2, b as the coefficient of the linear term x, and c as the constant term.

The function f can also be defined by the equivalent expression $(x + 2)(x + 1)$. When the quadratic expression is a product of two factors where each one is a linear expression, this is called the **factored form**.

An expression in factored form can be rewritten in standard form by expanding it, which means multiplying out the factors. In a previous lesson we saw how to use a diagram and to apply the distributive property to multiply two linear expressions, such as $(x + 3)(x + 2)$. We can do the same to expand an expression with a sum and a difference, such as $(x + 5)(x - 2)$, or to expand an expression with two differences, for example, $(x - 4)(x - 1)$.

To represent $(x - 4)(x - 1)$ with a diagram, we can think of subtraction as adding the opposite:

	x	-4
x	x^2	$-4x$
-1	$-x$	4

$$\begin{aligned}
&(x - 4)(x - 1)\\
&= (x + -4)(x + -1)\\
&= x\,(x + -1) + -4\,(x + -1)\\
&= x^2 + -1x + -4x + (-4)(-1)\\
&= x^2 + -5x + 4\\
&= x^2 - 5x + 4
\end{aligned}$$

Lesson 9: Standard Form and Factored Form

Cool Down: From One Form to Another

For each expression, write an equivalent expression in standard form. Show your reasoning.

1. $(2x+5)(x+1)$

2. $(x-2)(x+2)$

Unit 6 Lesson 9 Cumulative Practice Problems

1. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x+4)(x-1)$

   b. $(2x-1)(3x-1)$

2. Consider the expression $8-6x+x^2$.

   a. Is the expression in standard form? Explain how you know.

   b. Is the expression equivalent to $(x-4)(x-2)$? Explain how you know.

3. Which quadratic expression is written in standard form?

   A. $(x+3)x$

   B. $(x+4)^2$

   C. $-x^2-5x+7$

   D. $x^2+2(x+3)$

4. Explain why $3x^2$ can be said to be in both standard form and factored form.

5. Jada dropped her sunglasses from a bridge over a river. Which equation could represent the distance y fallen in feet as a function of time, t, in seconds?

   A. $y = 16t^2$

   B. $y = 48t$

   C. $y = 180 - 16t^2$

   D. $y = 180 - 48t$

   (From Unit 6, Lesson 5.)

6. A football player throws a football. The function h given by $h(t) = 6 + 75t - 16t^2$ describes the football's height in feet t seconds after it is thrown.

   Select **all** the statements that are true about this situation.

   A. The football is thrown from ground level.

   B. The football is thrown from 6 feet off the ground.

   C. In the function, $-16t^2$ represents the effect of gravity.

   D. The outputs of h decrease then increase in value.

   E. The function h has 2 zeros that make sense in this situation.

   F. The vertex of the graph of h gives the maximum height of the football.

   (From Unit 6, Lesson 6.)

7. *Technology required*. Two rocks are launched straight up in the air.

   - The height of Rock A is given by the function f, where $f(t) = 4 + 30t - 16t^2$.
   - The height of Rock B is given by function g, where $g(t) = 5 + 20t - 16t^2$.

   In both functions, t is time measured in seconds and height is measured in feet. Use graphing technology to graph both equations.

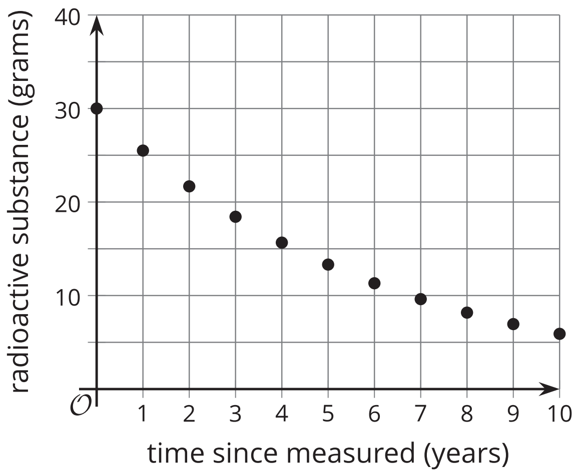
   a. What is the maximum height of each rock?

   b. Which rock reaches its maximum height first? Explain how you know.

   (From Unit 6, Lesson 6.)

8. The graph shows the number of grams of a radioactive substance in a sample at different times after the sample was first analyzed.

   a. What is the average rate of change for the substance during the 10 year period?



   b. Is the average rate of change a good measure for the change in the radioactive substance during these 10 years? Explain how you know.

   (From Unit 5, Lesson 10.)

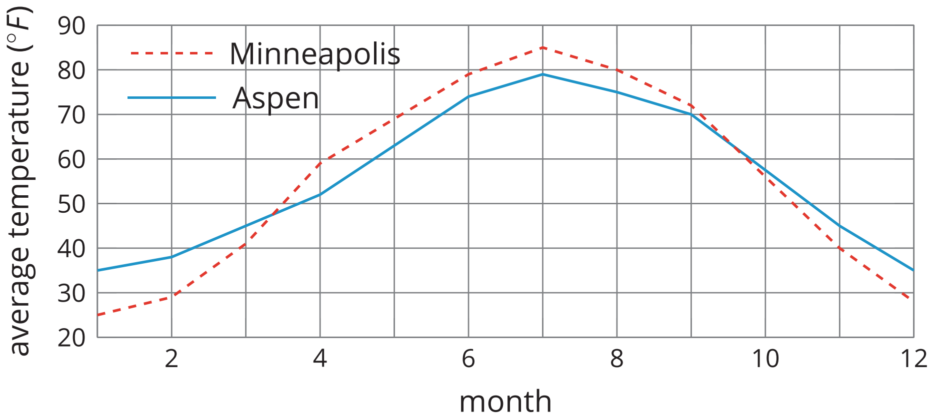
9. Each day after an outbreak of a new strain of the flu virus, a public health scientist receives a report of the number of new cases of the flu reported by area hospitals.

time since outbreak in days	2	3	4	5	6	7
number of new cases of the flu	20	28	38	54	75	105

Would a linear or exponential model be more appropriate for this data? Explain how you know.

(From Unit 5, Lesson 11.)

10. $A(t)$ is the average high temperature in Aspen, Colorado, t months after the start of the year. $M(t)$ is the temperature in Minneapolis, Minnesota, t months after the start of the year. Temperature is measured in degrees Fahrenheit.



a. What does $A(8)$ mean in this situation? Estimate $A(8)$.

b. Which city had a higher average temperature in February?

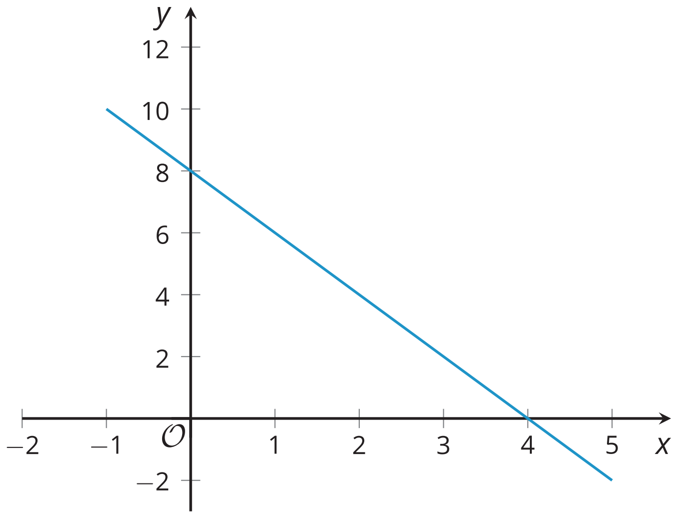
c. Were the two cities' average high temperatures ever the same? If so, when?

(From Unit 4, Lesson 9.)

Lesson 10: Graphs of Functions in Standard and Factored Forms

10.1: A Linear Equation and Its Graph

Here is a graph of the equation $y = 8 - 2x$.



1. Where do you see the 8 from the equation in the graph?

2. Where do you see the -2 from the equation in the graph?

3. What is the x-intercept of the graph? How does this relate to the equation?

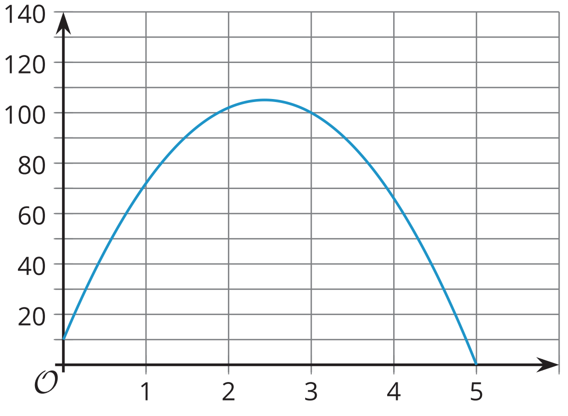
10.2: Revisiting Projectile Motion

In an earlier lesson, we saw that an equation such as $h(t) = 10 + 78t - 16t^2$ can model the height of an object thrown upward from a height of 10 feet with a vertical velocity of 78 feet per second.

1. Is the expression $10 + 78t - 16t^2$ written in standard form? Explain how you know.

2. Jada said that the equation $g(t) = (\text{-}16t - 2)(t - 5)$ also defines the same function, written in factored form. Show that Jada is correct.

3. Here is a graph representing both $g(t) = (\text{-}16t - 2)(t - 5)$ and $h(t) = 10 + 78t - 16t^2$.



a. Identify or approximate the vertical and horizontal intercepts.

b. What do each of these points mean in this situation?

10.3: Relating Expressions and Their Graphs

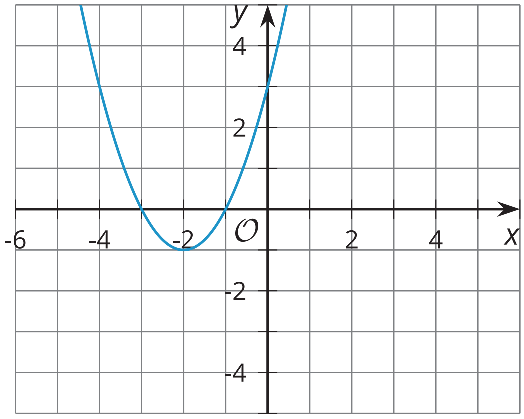
Here are pairs of expressions in standard and factored forms. Each pair of expressions define the same quadratic function, which can be represented with the given graph.

1. Identify the x-intercepts and the y-intercept of each graph.

Function f

$x^2 + 4x + 3$

$(x + 3)(x + 1)$



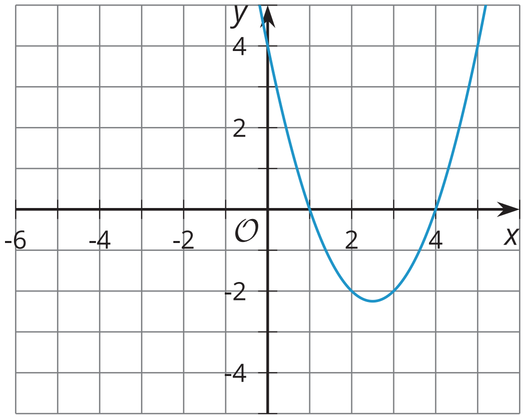
x-intercepts:

y-intercept:

Function g

$x^2 - 5x + 4$

$(x - 4)(x - 1)$



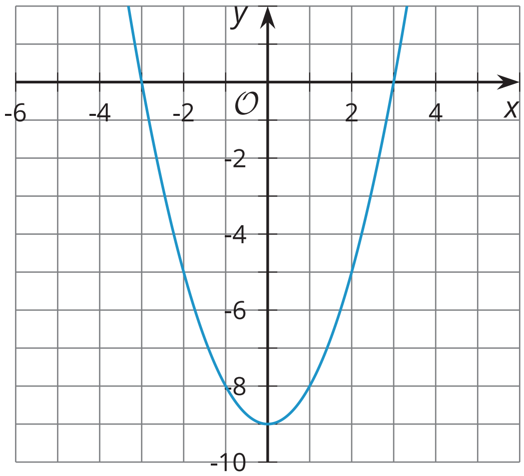
x-intercepts:

y-intercept:

Function h

$x^2 - 9$

$(x - 3)(x + 3)$



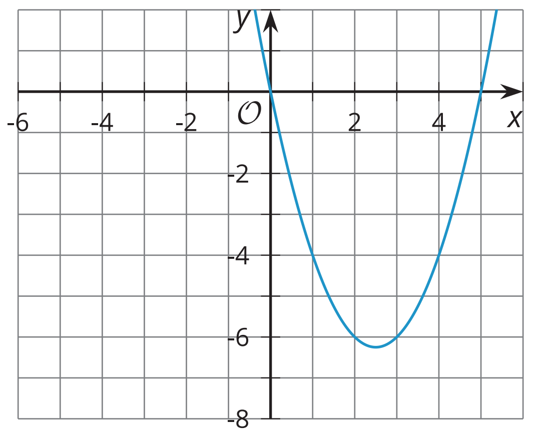
x-intercepts:

y-intercept:

Function i

$x^2 - 5x$

$x(x-5)$



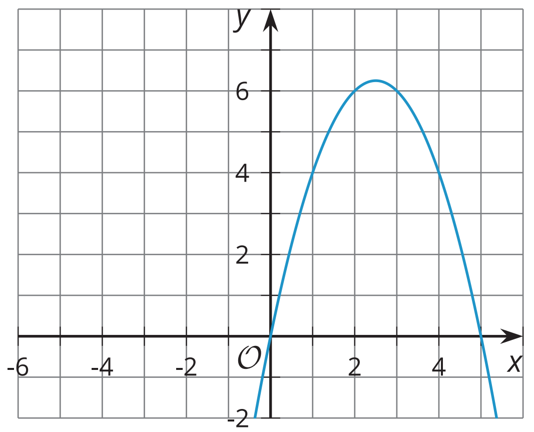
x-intercepts:

y-intercept:

Function j

$5x - x^2$

$x(5-x)$



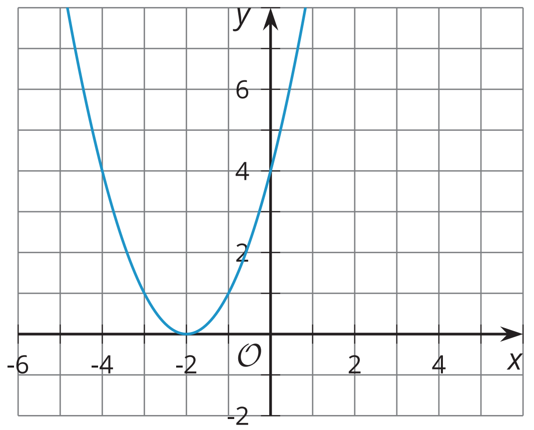
x-intercepts:

y-intercept:

Function k

$x^2 + 4x + 4$

$(x+2)(x+2)$



x-intercepts:

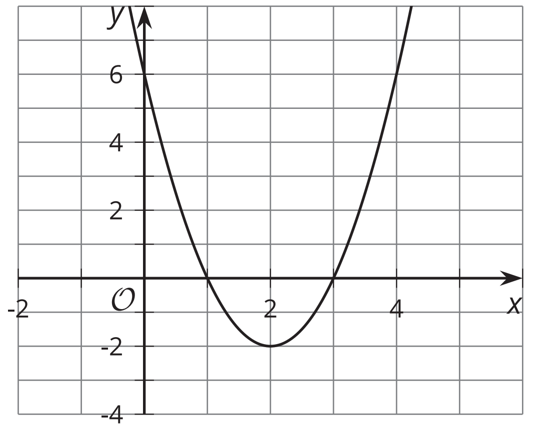
y-intercept:

2. What do you notice about the x-intercepts, the y-intercept, and the numbers in the expressions defining each function? Make a couple of observations.

3. Here is an expression that models function p, another quadratic function: $(x-9)(x-1)$. Predict the x-intercepts and the y-intercept of the graph that represent this function.

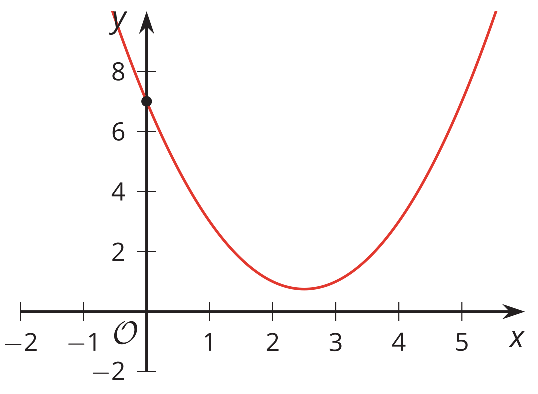
Are you ready for more?

Find the values of a, p, and q that will make $y = a(x-p)(x-q)$ be the equation represented by the graph.



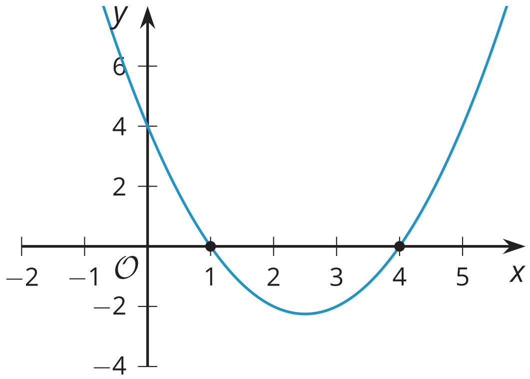
Lesson 10 Summary

Different forms of quadratic functions can tell us interesting information about the function's graph. When a quadratic function is expressed in standard form, it can tell us the y-intercept of the graph representing the function. For example, the graph representing $y = x^2 - 5x + 7$ has its y-intercept $(0, 7)$. This makes sense because the y-coordinate is the y-value when x is 0. Evaluating the expression at $x = 0$ gives $y = 0^2 - 5(0) + 7$, which equals 7.

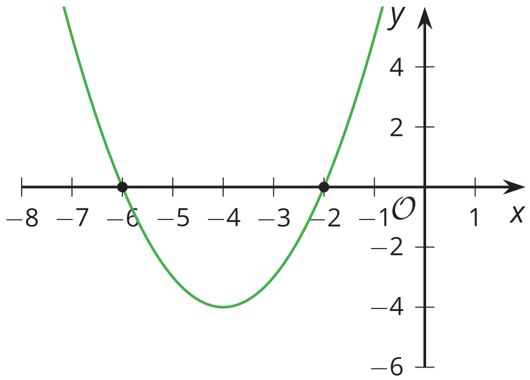


When a function is expressed in factored form, it can help us see the x-intercepts of its graph. Let's look at the functions f given by $f(x) = (x-4)(x-1)$ and g given by $g(x) = (x+2)(x+6)$.

If we graph $y = f(x)$, we see that the x-intercepts of the graph are $(1, 0)$ and $(4, 0)$. Notice that "1" and "4" also appear in $f(x) = (x-4)(x-1)$, and they are subtracted from x.



If we graph $y = g(x)$, we see that the x-intercepts are at $(\text{-}2, 0)$ and $(\text{-}6, 0)$. Notice that "2" and "6" are also in the equation $g(x) = (x+2)(x+6)$, but they are added to x.



The connection between the factored form and the x-intercepts of the graph tells us about the zeros of the function (the input values that produce an output of 0). In the next lesson, we will further explore these connections between different forms of quadratic expressions and the graphs representing them.

Lesson 10: Graphs of Functions in Standard and Factored Forms

Cool Down: Making Connections

The equations $y = x^2 + 6x + 8$ and $y = (x + 2)(x + 4)$ both define the same quadratic function. Without graphing, identify the x- and y-intercepts of the graph. Explain how you know.

Unit 6 Lesson 10 Cumulative Practice Problems

1. A quadratic function f is defined by $f(x) = (x - 7)(x + 3)$.

    a. Without graphing, identify the x-intercepts of the graph of f. Explain how you know.

    b. Expand $(x - 7)(x + 3)$ and use the expanded form to identify the y-intercept of the graph of f.

2. What are the x-intercepts of the graph of the function defined by $(x - 2)(2x + 1)$?

    A. $(2, 0)$ and $(-1, 0)$

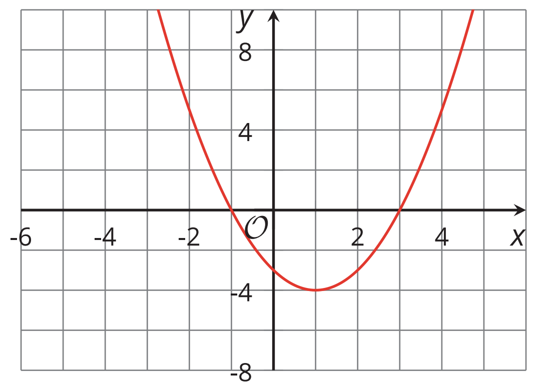
    B. $(2, 0)$ and $\left(-\frac{1}{2}, 0\right)$

    C. $(-2, 0)$ and $(1, 0)$

    D. $(-2, 0)$ and $(\frac{1}{2}, 0)$

3. Here is a graph that represents a quadratic function.

    Which expression could define this function?



    A. $(x + 3)(x + 1)$

    B. $(x + 3)(x - 1)$

    C. $(x - 3)(x + 1)$

    D. $(x - 3)(x - 1)$

4. a. What is the y-intercept of the graph of the equation $y = x^2 - 5x + 4$?

   b. An equivalent way to write this equation is $y = (x - 4)(x - 1)$. What are the x-intercepts of this equation's graph?

5. Noah said that if we graph $y = (x - 1)(x + 6)$, the x-intercepts will be at $(1, 0)$ and $(\text{-}6, 0)$. Explain how you can determine, without graphing, whether Noah is correct.

6. A company sells a video game. If the price of the game in dollars is p the company estimates that it will sell $20{,}000 - 500p$ games.

   Which expression represents the revenue in dollars from selling games if the game is priced at p dollars?

   A. $(20{,}000 - 500p) + p$

   B. $(20{,}000 - 500p) - p$

   C. $\dfrac{20,000 - 500p}{p}$

   D. $(20{,}000 - 500p) \cdot p$

   (From Unit 6, Lesson 7.)

7. Write each quadratic expression in standard form. Draw a diagram if needed.

   a. $(x - 3)(x - 6)$

   b. $(x - 4)^2$

   c. $(2x + 3)(x - 4)$

   d. $(4x - 1)(3x - 7)$

   (From Unit 6, Lesson 9.)

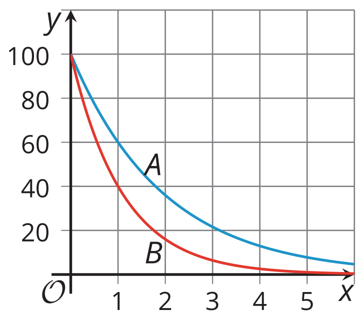
8. Consider the expression $(5+x)(6-x)$.

    a. Is the expression equivalent to x^2+x+30? Explain how you know.

    b. Is the expression $30+x-x^2$ in standard form? Explain how you know.

(From Unit 6, Lesson 9.)

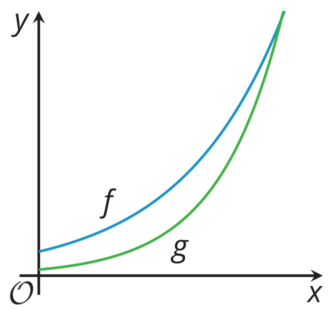
9. Here are graphs of the functions f and g given by $f(x)=100 \cdot \left(\frac{3}{5}\right)^x$ and $g(x)=100 \cdot \left(\frac{2}{5}\right)^x$.

    Which graph corresponds to f and which graph corresponds to g? Explain how you know.

(From Unit 5, Lesson 12.)

10. Here are graphs of two functions f and g.

    An equation defining f is $f(x)=100 \cdot 2^x$.

    Which of these could be an equation defining the function g?

    A. $g(x)=25 \cdot 3^x$

    B. $g(x)=50 \cdot (1.5)^x$

    C. $g(x)=100 \cdot 3^x$

    D. $g(x)=200 \cdot (1.5)^x$

(From Unit 5, Lesson 13.)