Illustrative Mathematics

Algebra 1
Introduction to Quadratics

Lessons 11-17

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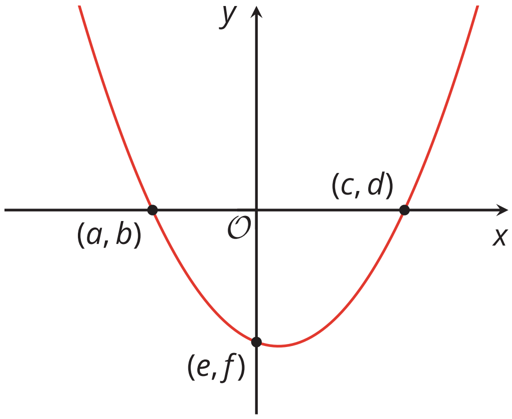
Illustrative Mathematics

Lesson 11: Graphing from the Factored Form

11.1: Finding Coordinates



Here is a graph of a function w defined by $w(x) = (x + 1.6)(x - 2)$. Three points on the graph are labeled.

Find the values of a, b, c, d, e, and f. Be prepared to explain your reasoning.

11.2: Comparing Two Graphs

Consider two functions defined by $f(x) = x(x+4)$ and $g(x) = x(x-4)$.

1. Complete the table of values for each function. Then, determine the x-intercepts and vertex of each graph. Be prepared to explain how you know.

x	$f(x)$
-5	5
-4	
-3	
-2	-4
-1	-3
0	
1	
2	
3	
4	32
5	

x-intercepts:

Vertex:

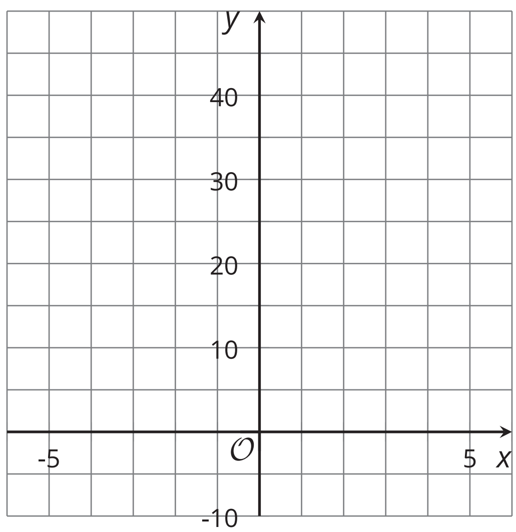
x	$g(x)$
-5	45
-4	
-3	
-2	12
-1	5
0	
1	
2	
3	-3
4	
5	

x-intercepts:

Vertex:

2. Plot the points from the tables on the same coordinate plane. (Consider using different colors or markings for each set of points so you can tell them apart.)

   Then, make a couple of observations about how the two graphs compare.



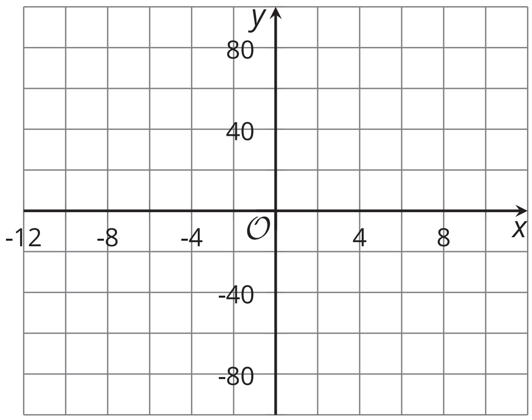
11.3: What Do We Need to Sketch a Graph?

1. The functions f, g, and h are given. Predict the x-intercepts and the x-coordinate of the vertex of each function.

equation	x-intercepts	x-coordinate of the vertex
$f(x) = (x + 3)(x - 5)$		
$g(x) = 2x(x - 3)$		
$h(x) = (x + 4)(4 - x)$		

2. Use graphing technology to graph the functions f, g, and h. Use the graphs to check your predictions.

3. Without using technology, sketch a graph that represents the equation $y = (x - 7)(x + 11)$ and that shows the x-intercepts and the vertex. Think about how to find the y-coordinate of the vertex. Be prepared to explain your reasoning.



Are you ready for more?

The quadratic function f is given by $f(x) = x^2 + 2x + 6$.

1. Find $f(\text{-}2)$ and $f(0)$.

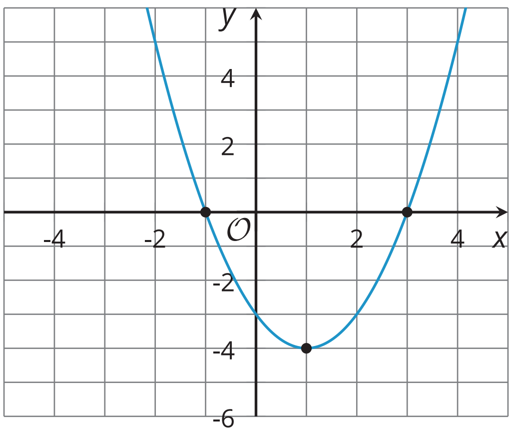
2. What is the x-coordinate of the vertex of the graph of this quadratic function?

3. Does the graph have any x-intercepts? Explain or show how you know.

Lesson 11 Summary

The function f given by $f(x) = (x+1)(x-3)$ is written in factored form. Recall that this form is helpful for finding the zeros of the function (where the function has the value 0) and telling us the x-intercepts on the graph representing the function.

Here is a graph representing f. It shows 2 x-intercepts at $x = \text{-}1$ and $x = 3$.



If we use -1 and 3 as inputs to f, what are the outputs?

- $f(\text{-}1) = (\text{-}1+1)(\text{-}1-3) = (0)(\text{-}4) = 0$
- $f(3) = (3+1)(3-3) = (4)(0) = 0$

Because the inputs -1 and 3 produce an output of 0, they are the zeros of the function f. And because both x values have 0 for their y value, they also give us the x-intercepts of the graph (the points where the graph crosses the x-axis, which always have a y-coordinate of 0). So, the zeros of a function have the same values as the x-coordinates of the x-intercepts of the graph of the function.

The factored form can also help us identify the vertex of the graph, which is the point where the function reaches its minimum value. Notice that the x-coordinate of the vertex is 1, and that 1 is halfway between -1 and 3. Once we know the x-coordinate of the vertex, we can find the y-coordinate by evaluating the function: $f(1) = (1+1)(1-3) = 2(\text{-}2) = \text{-}4$. So the vertex is at $(1, \text{-}4)$.

When a quadratic function is in standard form, the y-intercept is clear: its y-coordinate is the constant term c in $ax^2 + bx + c$. To find the y-intercept from factored form, we can evaluate the function at $x = 0$, because the y-intercept is the point where the graph has an input value of 0. $f(0) = (0+1)(0-3) = (1)(\text{-}3) = \text{-}3$.

Lesson 11: Graphing from the Factored Form

Cool Down: Sketching a Graph

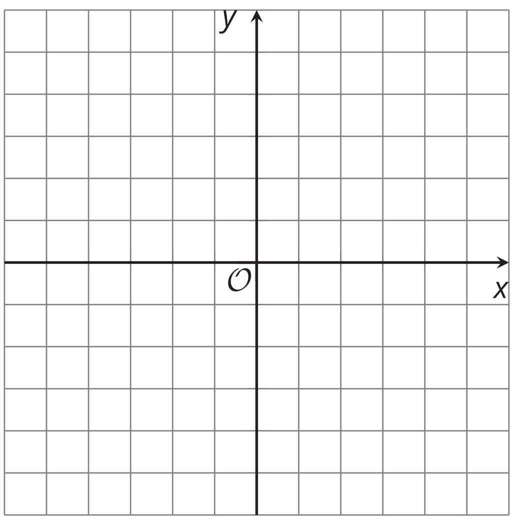
The function f is given by $f(x) = (x - 2)(x + 4)$. Without using graphing technology, answer the following questions.

1. What are the x-intercepts of the graph representing f?

2. What are the x- and y-coordinates of the vertex of the graph?

3. What is the y-intercept?

4. Sketch a graph that represents f.



Unit 6 Lesson 11 Cumulative Practice Problems

1. Select **all** true statements about the graph that represents $y = 2x(x - 11)$.

   A. Its x-intercepts are at $(-2, 0)$ and $(11, 0)$.

   B. Its x-intercepts are at $(0, 0)$ and $(11, 0)$.

   C. Its x-intercepts are at $(2, 0)$ and $(-11, 0)$.

   D. It has only one x-intercept.

   E. The x-coordinate of its vertex is -4.5.

   F. The x-coordinate of its vertex is 11.

   G. The x-coordinate of its vertex is 4.5.

   H. The x-coordinate of its vertex is 5.5.

2. Select **all** equations whose graphs have a vertex with x-coordinate 2.

   A. $y = (x - 2)(x - 4)$

   B. $y = (x - 2)(x + 2)$

   C. $y = (x - 1)(x - 3)$

   D. $y = x(x + 4)$

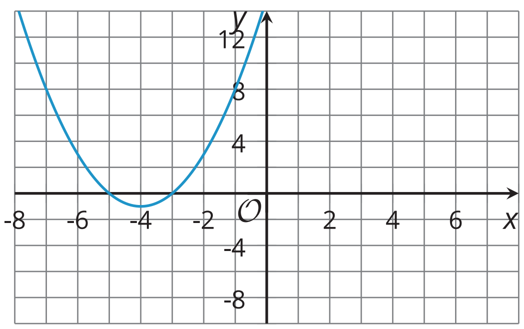
   E. $y = x(x - 4)$

3. Determine the x-intercepts and the x-coordinate of the vertex of the graph that represents each equation.

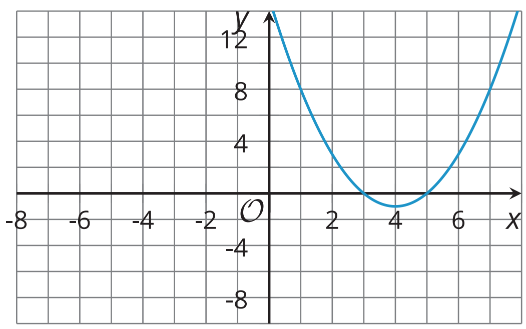
equation	x-intercepts	x-coordinate of the vertex
$y = x(x - 2)$		
$y = (x - 4)(x + 5)$		
$y = \text{-}5x(3 - x)$		

4. Which one is the graph of the equation $y = (x - 3)(x + 5)$?

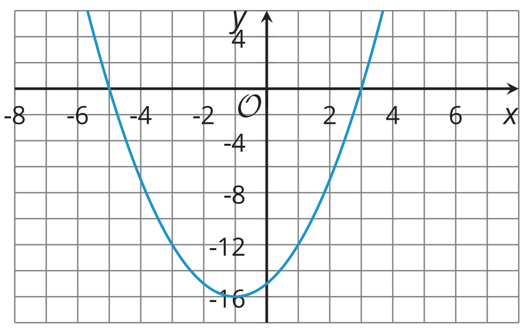
Graph A



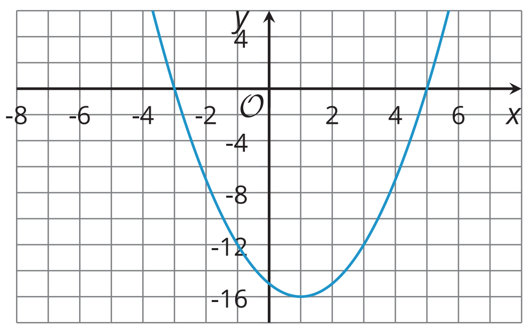
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

5. a. What are the x-intercepts of the graph of $y = (x-2)(x-4)$?

   b. Find the coordinates of another point on the graph. Show your reasoning.

   c. Sketch a graph of the equation $y = (x-2)(x-4)$.

6. A company sells calculators. If the price of the calculator in dollars is p, the company estimates that it will sell $10{,}000 - 120p$ calculators.

   Write an expression that represents the revenue in dollars from selling calculators if a calculator is priced at p dollars.

   (From Unit 6, Lesson 7.)

7. Is $(s+t)^2$ equivalent to $s^2 + 2st + t^2$? Explain or show your reasoning.

   (From Unit 6, Lesson 8.)

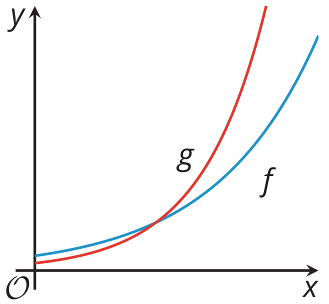
8. Tyler is shopping for a truck. He found two trucks that he likes. One truck sells for \$7,200. A slightly older truck sells for 15% less. How much does the older truck cost?

   (From Unit 5, Lesson 14.)

9. Here are graphs of two exponential functions, f and g.

   The function f is given by $f(x) = 100 \cdot 2^x$ while g is given by $g(x) = a \cdot b^x$.

   Based on the graphs of the functions, what can you conclude about a and b?



   (From Unit 5, Lesson 13.)

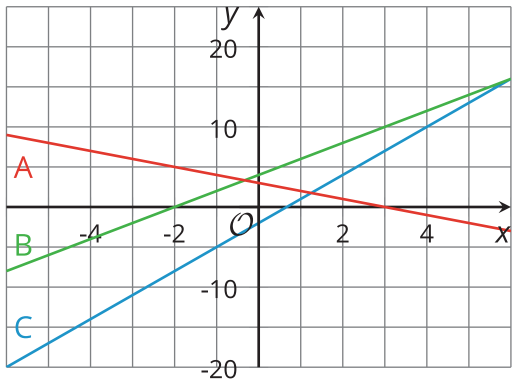
10. Suppose G takes a student's grade and gives a student's name as the output. Explain why G is not a function.

    (From Unit 4, Lesson 2.)

Lesson 12: Graphing the Standard Form (Part 1)

12.1: Matching Graphs to Linear Equations

Graphs A, B, and C represent 3 linear equations: $y = 2x + 4$, $y = 3 - x$, and $y = 3x - 2$. Which graph corresponds to which equation? Explain your reasoning.



12.2: Quadratic Graphs Galore

Using graphing technology, graph $y = x^2$, and then experiment with each of the following changes to the function. Record your observations (include sketches, if helpful).

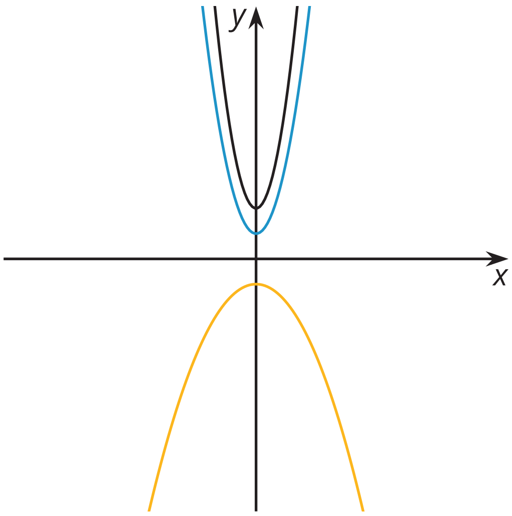
1. Adding different constant terms to x^2 (for example: $x^2 + 5$, $x^2 + 10$, $x^2 - 3$, etc.)

2. Multiplying x^2 by different positive coefficients greater than 1 (for example: $3x^2$, $7.5x^2$, etc.)

3. Multiplying x^2 by different negative coefficients less than -1 (for example: -x^2, -$4x^2$, etc.)

4. Multiplying x^2 by different coefficients between -1 and 1 (for example: $\frac{1}{2}x^2$, -$0.25x^2$, etc.)

Are you ready for more?



Here are the graphs of three quadratic functions. What can you say about the coefficients of x^2 in the expressions that define f, g, and h? Can you identify them? How do they compare?

12.3: What Do These Tables Reveal?

1. a. Complete the table with values of $x^2 + 10$ and $x^2 - 3$ at different values of x. (You may also use a spreadsheet tool, if available.)

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 10$							
$x^2 - 3$							

   b. Earlier, you observed the effects on the graph of adding or subtracting a constant term from x^2. Study the values in the table. Use them to explain why the graphs changed they way they did when a constant term is added or subtracted.

2. a. Complete the table with values of $2x^2$, $\frac{1}{2}x^2$, and $-2x^2$ at different values of x. (You may also use a spreadsheet tool, if available.)

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$							
$\frac{1}{2}x^2$							
$-2x^2$							

   b. You also observed the effects on the graph of multiplying x^2 by different coefficients. Study the values in the table. Use them to explain why the graphs changed they way they did when x^2 is multiplied by a number greater than 1, by a negative number less than -1, and by numbers between -1 and 1.

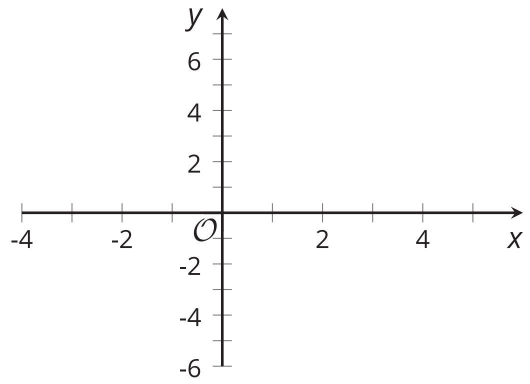
12.4: Card Sort: Representations of Quadratic Functions

Your teacher will give your group a set of cards. Each card contains a graph or an equation.

- Take turns with your partner to sort the cards into sets so that each set contains two equations and a graph that all represent the same quadratic function.
- For each set of cards that you put together, explain to your partner how you know they belong together.
- For each set that your partner puts together, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are sorted and discussed, record the equivalent equations, sketch the corresponding graph, and write a brief note or explanation about why the representations were grouped together.

Standard form:

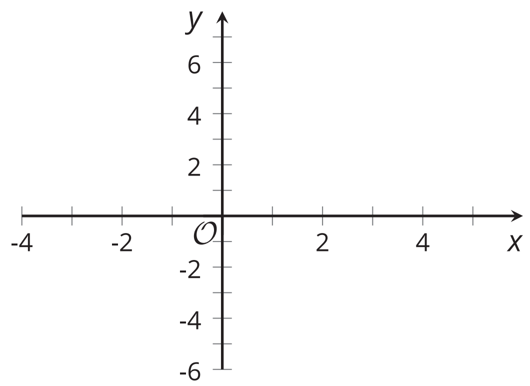
Factored form:



Explanation:

Standard form:

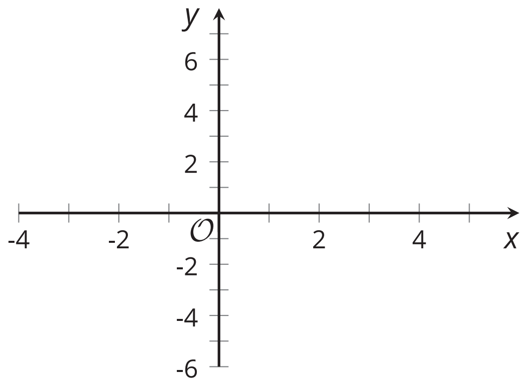
Factored form:



Explanation:

Standard form:

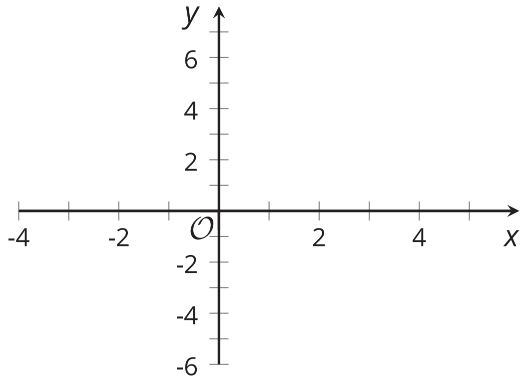
Factored form:



Explanation:

Standard form:

Factored form:

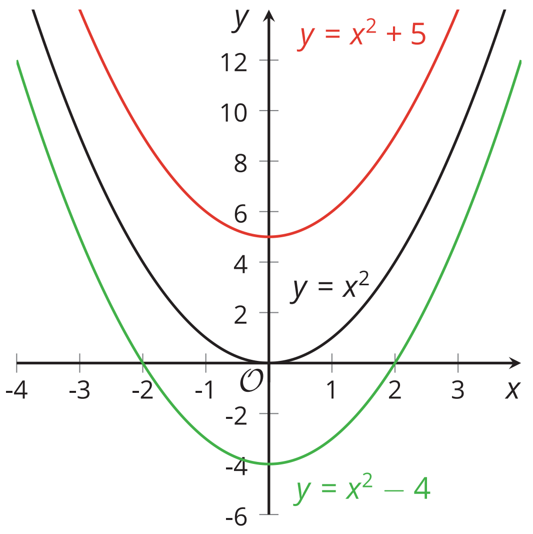


Explanation:

Lesson 12 Summary

Remember that the graph representing any quadratic function is a shape called a *parabola*. People often say that a parabola “opens upward” when the lowest point on the graph is the vertex (where the graph changes direction), and “opens downward” when the highest point on the graph is the vertex. Each coefficient in a quadratic expression written in standard form $ax^2 + bx + c$ tells us something important about the graph that represents it.

The graph of $y = x^2$ is a parabola opening upward with vertex at $(0, 0)$. Adding a constant term 5 gives $y = x^2 + 5$ and raises the graph by 5 units. Subtracting 4 from x^2 gives $y = x^2 - 4$ and moves the graph 4 units down.



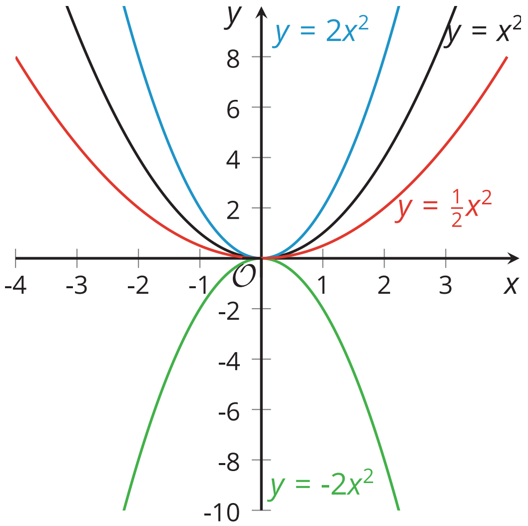
x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 5$	14	9	6	5	6	9	14
$x^2 - 4$	5	0	-3	-4	-3	0	5

A table of values can help us see that adding 5 to x^2 increases all the output values of $y = x^2$ by 5, which explains why the graph moves up 5 units. Subtracting 4 from x^2 decreases all the output values of $y = x^2$ by 4, which explains why the graph shifts down by 4 units.

In general, the constant term of a quadratic expression in standard form influences the vertical position of the graph. An expression with no constant term (such as x^2 or $x^2 + 9x$) means that the constant term is 0, so the y-intercept of the graph is on the x-axis. It’s not shifted up or down relative to the x-axis.

The coefficient of the squared term in a quadratic function also tells us something about its graph. The coefficient of the squared term in $y = x^2$ is 1. Its graph is a parabola that opens upward.

- Multiplying x^2 by a number greater than 1 makes the graph steeper, so the parabola is narrower than that representing x^2.
- Multiplying x^2 by a number less than 1 but greater than 0 makes the graph less steep, so the parabola is wider than that representing x^2.
- Multiplying x^2 by a number less than 0 makes the parabola open downward.



x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$2x^2$	18	8	2	0	2	8	18
$-2x^2$	-18	-4	-2	0	-2	-8	-18

If we compare the output values of $2x^2$ and $-2x^2$, we see that they are opposites, which suggests that one graph would be a reflection of the other across the x-axis.

Card Sort: Representations of Quadratic Functions

$$y = x^2 - 1$$

Card Sort: Representations of Quadratic Functions

$$y = x(x-4)$$

Card Sort: Representations of Quadratic Functions

$$y = x^2 - 4x + 4$$

Card Sort: Representations of Quadratic Functions

$$y = (x+1)(x-1)$$

Card Sort: Representations of Quadratic Functions

$$y = (x-1)(x-4)$$

Card Sort: Representations of Quadratic Functions

$$y = x^2 - 4x$$

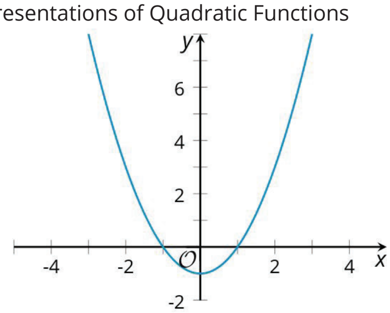
Card Sort: Representations of Quadratic Functions

$$y = (x-2)^2$$

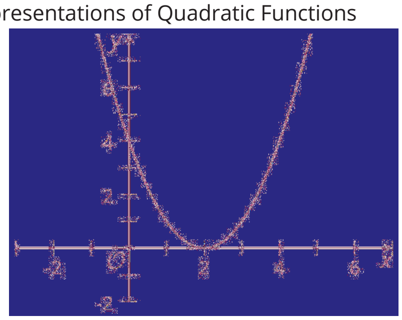
Card Sort: Representations of Quadratic Functions

$$y = x^2 - 5x + 4$$

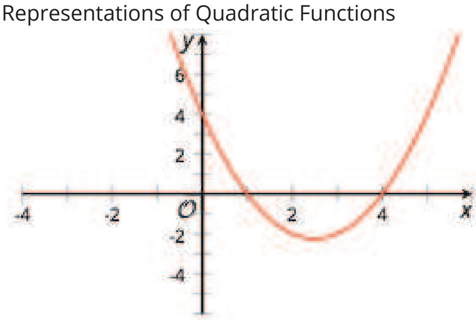
Card Sort: Representations of Quadratic Functions



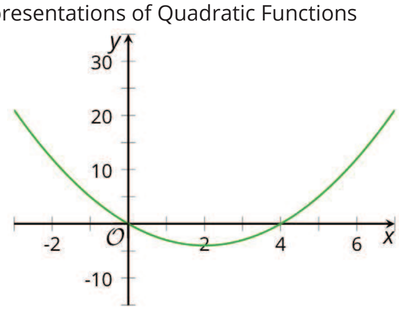
Card Sort: Representations of Quadratic Functions



Card Sort: Representations of Quadratic Functions



Card Sort: Representations of Quadratic Functions

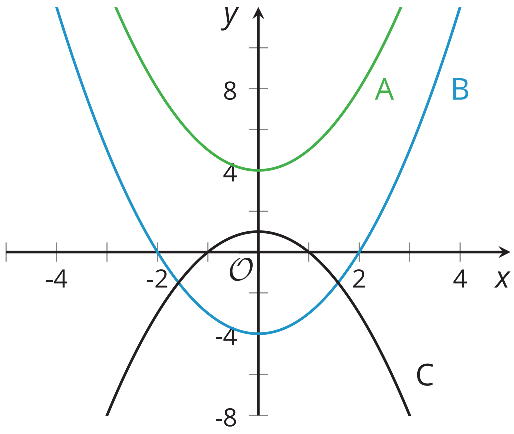


Lesson 12: Graphing the Standard Form (Part 1)

Cool Down: Matching Equations and Graphs

Here are graphs that represent three quadratic functions, defined by:

$$f(x) = x^2 - 4$$
$$g(x) = 1 - x^2$$
$$h(x) = x^2 + 4$$



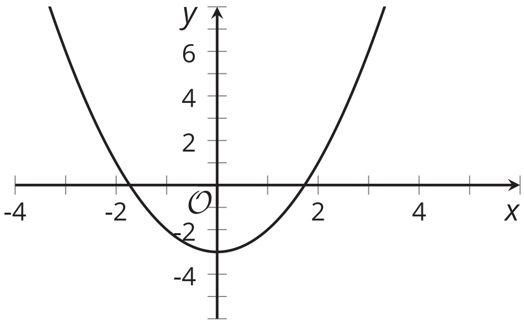
1. Match each equation to a graph that represents it. Explain how you know.

2. Write down the equation that can be represented by Graph C. Which part of the equation tells us that the graph opens downward?

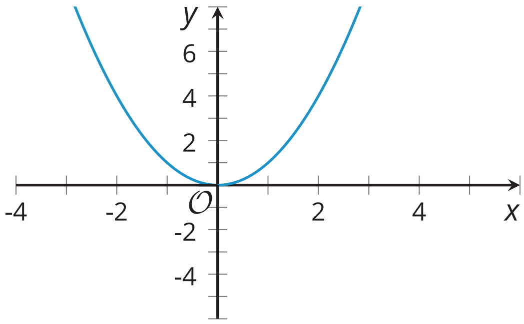
Unit 6 Lesson 12 Cumulative Practice Problems

1. Here are four graphs. Match each graph with a quadratic equation that it represents.

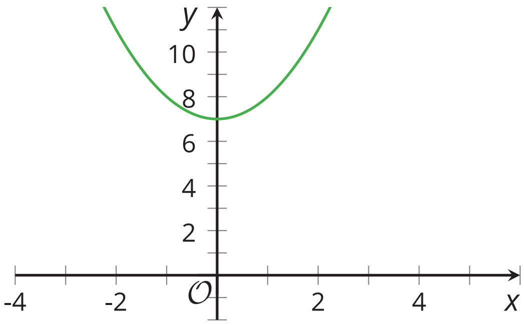
Graph A



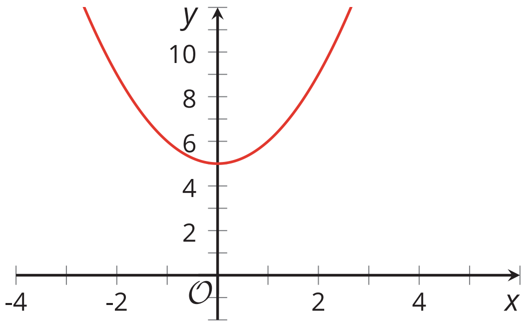
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. $y = x^2$
2. $y = x^2 + 5$
3. $y = x^2 + 7$
4. $y = x^2 - 3$

2. The two equations $y = (x + 2)(x + 3)$ and $y = x^2 + 5x + 6$ are equivalent.

   a. Which equation helps find the x-intercepts most efficiently?

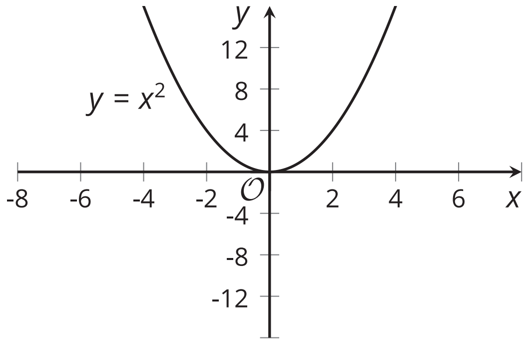
   b. Which equation helps find the y-intercept most efficiently?

3. Here is a graph that represents $y = x^2$.

   On the same coordinate plane, sketch and label the graph that represents each equation:

   a. $y = x^2 - 4$

   b. $y = -x^2 + 5$



4. Select **all** equations whose graphs have a y-intercept with a positive y-coordinate.

   A. $y = x^2 + 3x - 2$

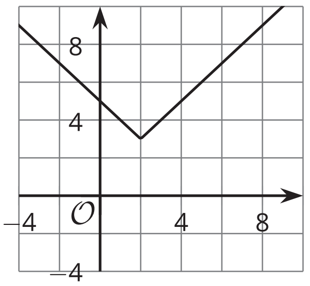
   B. $y = x^2 - 10x$

   C. $y = (x - 1)^2$

   D. $y = 5x^2 - 3x - 5$

   E. $y = (x + 1)(x + 2)$

5. a. Describe how the graph of $A(x) = |x|$ has to be shifted to match the given graph.

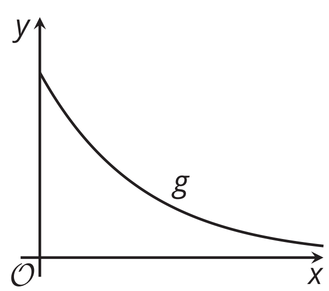
   

   b. Write an equation for the function represented by the graph.

   (From Unit 4, Lesson 14.)

6. Here is a graph of the function g given by $g(x) = a \cdot b^x$.

   What can you say about the value of b? Explain how you know.

   (From Unit 5, Lesson 13.)

7. a. What are the x-intercepts of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

   b. What is the x-coordinate of the vertex of the graph that represents $y = (x+1)(x+5)$? Explain how you know.

   c. Find the y-coordinate of the vertex. Show your reasoning.

   d. Sketch a graph of $y = (x+1)(x+5)$.

(From Unit 6, Lesson 11.)

8. Determine the x-intercepts, the vertex, and the y-intercept of the graph of each equation.

equation	x-intercepts	vertex	y-intercept
$y = (x-5)(x-3)$			
$y = 2x(8-x)$			

(From Unit 6, Lesson 11.)

9. Equal amounts of money were invested in stock A and stock B. In the first year, stock A increased in value by 20%, and stock B decreased by 20%. In the second year, stock A decreased in value by 20%, and stock B increased by 20%.

   Was one stock a better investment than the other? Explain your reasoning.

(From Unit 5, Lesson 15.)

Illustrative Mathematics

Lesson 13: Graphing the Standard Form (Part 2)

13.1: Equivalent Expressions

1. Complete each row with an equivalent expression in standard form or factored form.

standard form	factored form
x^2	
	$x(x+9)$
$x^2 - 18x$	
	$x(6-x)$
$-x^2 + 10x$	
	$-x(x+2.75)$

2. What do the quadratic expressions in each column have in common (besides the fact that everything in the left column is in standard form and everything in the other column is in factored form)? Be prepared to share your observations.

13.2: What about the Linear Term?

1. Using graphing technology:
    a. Graph $y = x^2$, and then experiment with adding different linear terms (for example, $x^2 + 4x$, $x^2 + 20x$, $x^2 - 50x$). Record your observations.

    b. Graph $y = -x^2$, and then experiment with adding different linear terms. Record your observations.

2. Use your observations to help you complete the table without graphing the equations.

equation	x-intercepts	x-coordinate of vertex
$y = x^2 + 6x$		
$y = x^2 - 10x$		
$y = -x^2 + 50x$		
$y = -x^2 - 36x$		

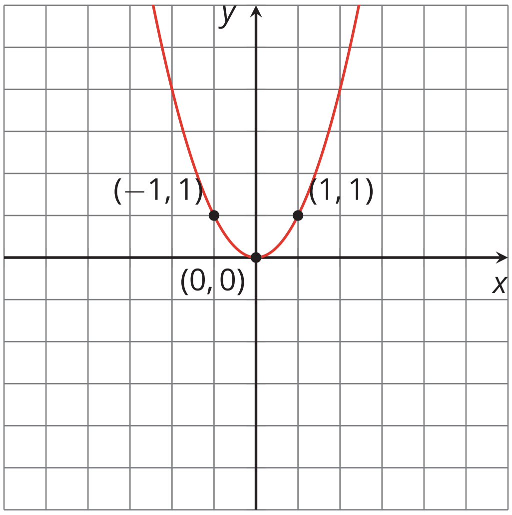
3. Some quadratic expressions have no linear terms. Find the x-intercepts and the x-coordinate of the vertex of the graph representing each equation. (Note it is possible for the graph to not intersect the x-axis.) If you get stuck, try graphing the equations.
   a. $y = x^2 - 25$

   b. $y = x^2 + 16$

13.3: Writing Equations to Match Graphs

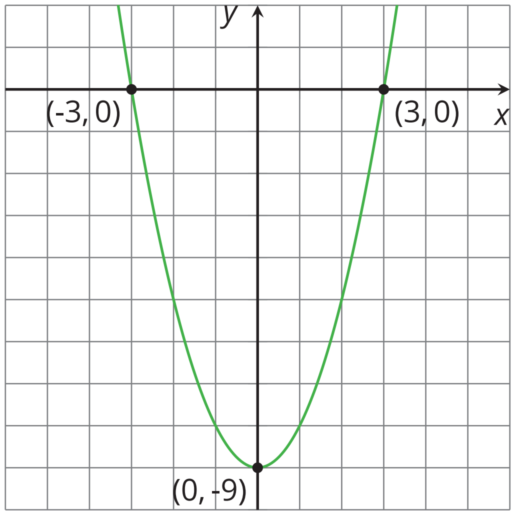
Use graphing technology to graph a function that matches each given graph. Make sure your graph goes through all 3 points shown!

A



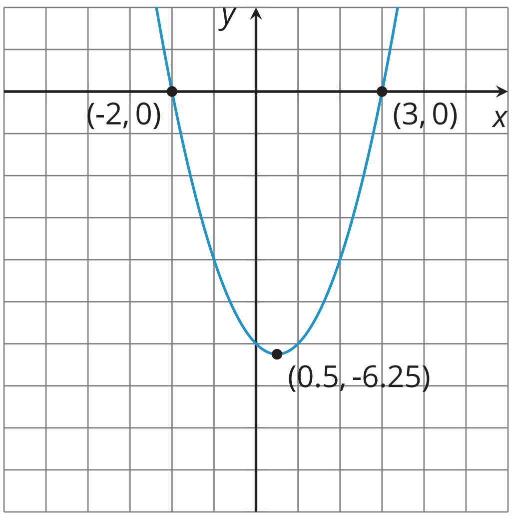
Equation:

B



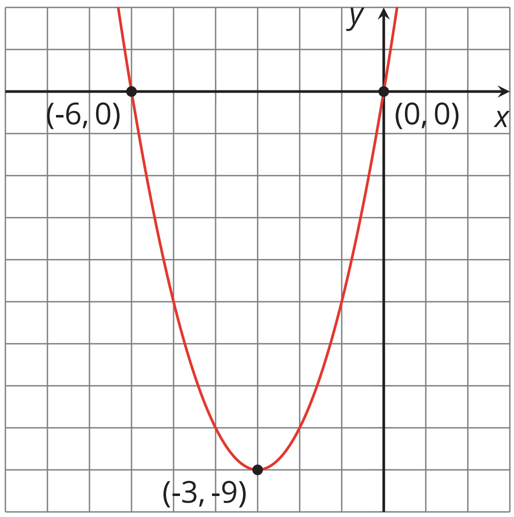
Equation:

C



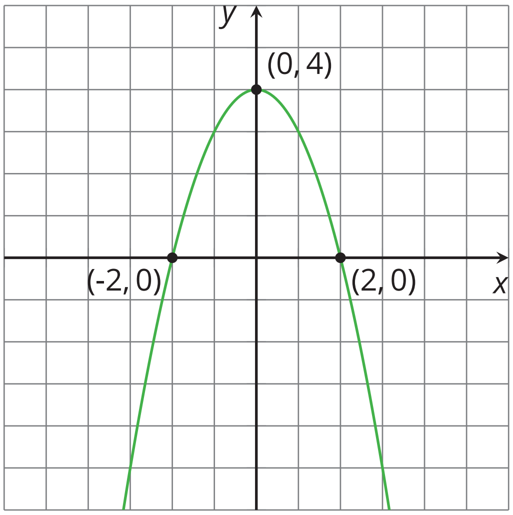
Equation:

D



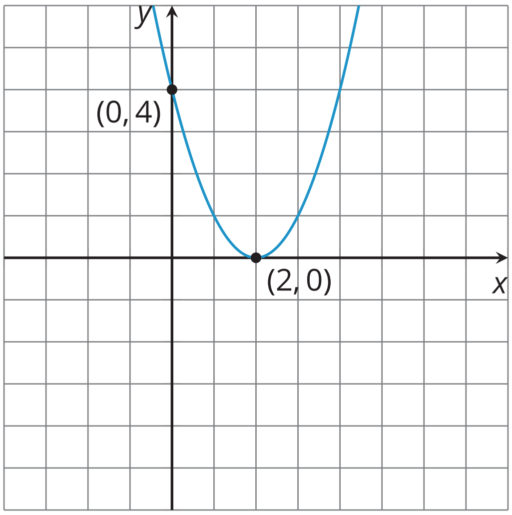
Equation:

E



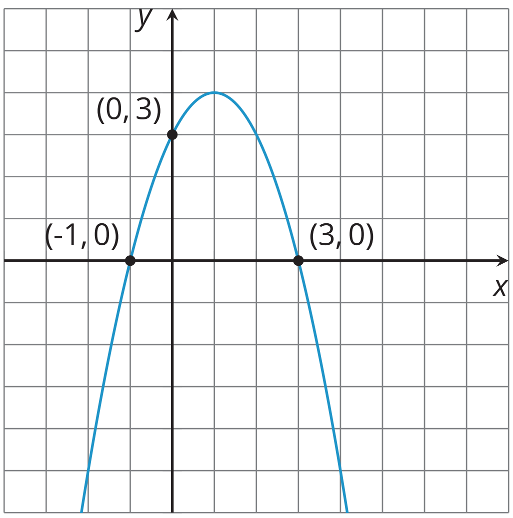
Equation:

F



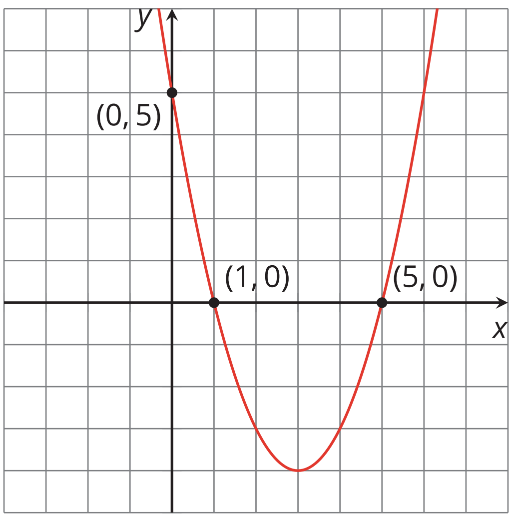
Equation:

G



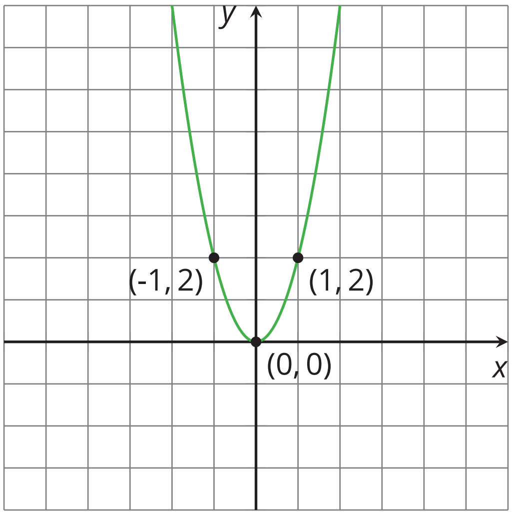
Equation:

H



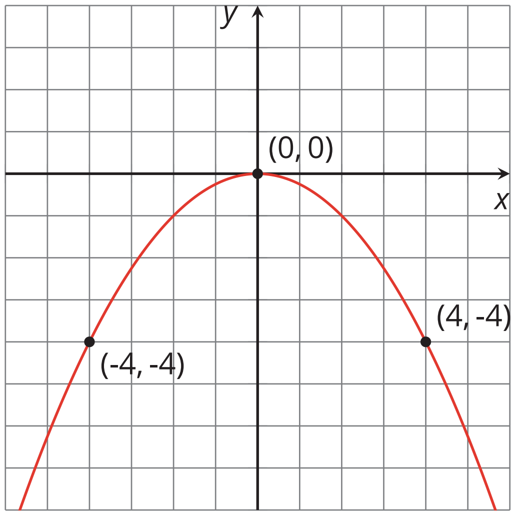
Equation:

I



Equation:

J



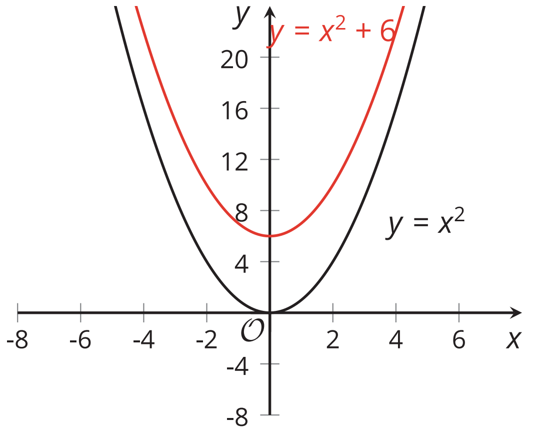
Equation:

Lesson 13 Summary

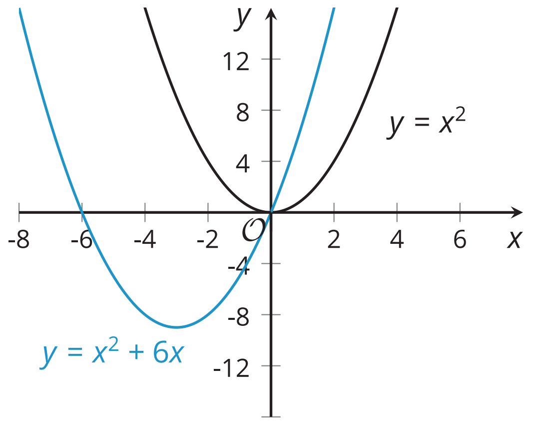
In an earlier lesson, we saw that a quadratic function written in standard form $ax^2 + bx + c$ can tell us some things about the graph that represents it. The coefficient a can tell us whether the graph of the function opens upward or downward, and also gives us information about whether it is narrow or wide. The constant term c can tell us about its vertical position.

Recall that the graph representing $y = x^2$ is an upward-opening parabola with the vertex at $(0, 0)$. The vertex is also the x-intercept and the y-intercept.

Suppose we add 6 to the squared term: $y = x^2 + 6$. Adding a 6 shifts the graph upwards, so the vertex is at $(0, 6)$. The vertex is the y-intercept and the graph is centered on the y-axis.



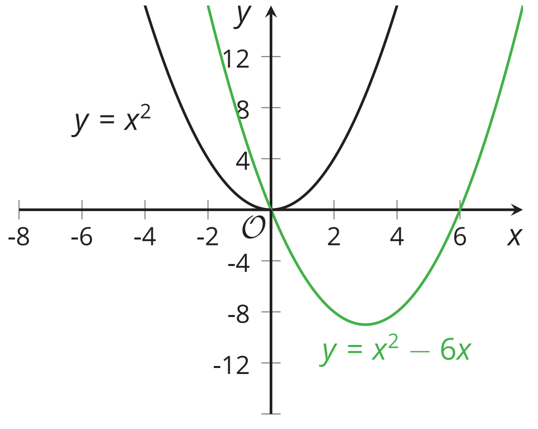
What can the linear term bx tell us about the graph representing a quadratic function?



The linear term has a somewhat mysterious effect on the graph of a quadratic function. The graph seems to shift both horizontally and vertically. When we add bx (where b is not 0) to x^2, the graph of $y = x^2 + bx$ is no longer centered on the y-axis.

Suppose we add $6x$ to the squared term: $y = x^2 + 6x$. Writing the $x^2 + 6x$ in factored form as $x(x + 6)$ gives us the zeros of the function, 0 and -6. Adding the term $6x$ seems to shift the graph to the left and down and the x-intercepts are now $(\text{-}6, 0)$ and $(0, 0)$. The vertex is no longer the y-intercept and the graph is no longer centered on the y-axis.

What if we add $\text{-}6x$ to x^2? $x^2 - 6x$ can be rewritten as $x(x - 6)$, which tells us the zeros: 0 and 6. Adding a negative linear term to a squared term seems to shift the graph to the right and down. The x-intercepts are now $(0, 0)$ and $(6, 0)$. The vertex is no longer the y-intercept and the graph is not centered on the y-axis.

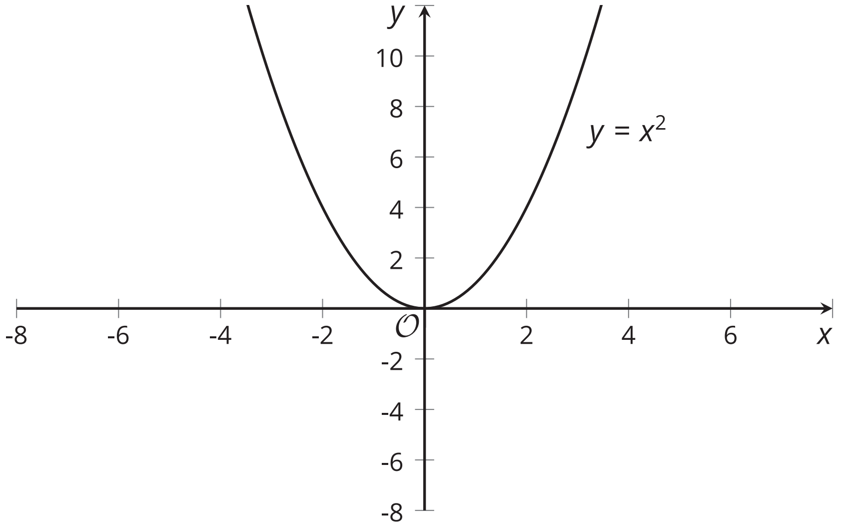


Lesson 13: Graphing the Standard Form (Part 2)

Cool Down: Sketching Graphs

1. Consider the quadratic equation $y = x^2 - 4x$. If we graph the equation, where are the x-intercepts located? What is the x-coordinate of the vertex?

2. Here is a graph of $y = x^2$. Sketch a graph of $y = x^2 + 5x$ on the same graph. Briefly explain how you know where to sketch the graph.

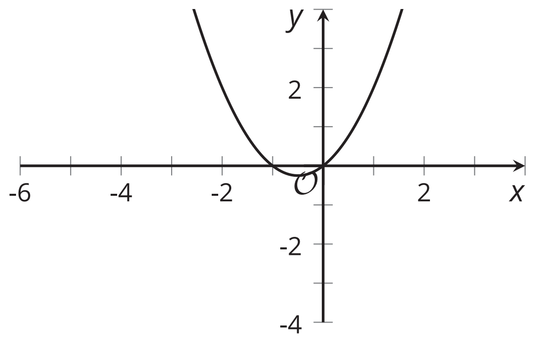


Unit 6 Lesson 13 Cumulative Practice Problems

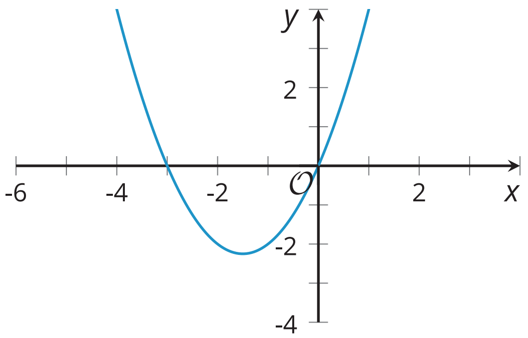
Unit 6 Lesson 13 Cumulative Practice Problems

1. Here are four graphs. Match each graph with the quadratic equation that it represents.

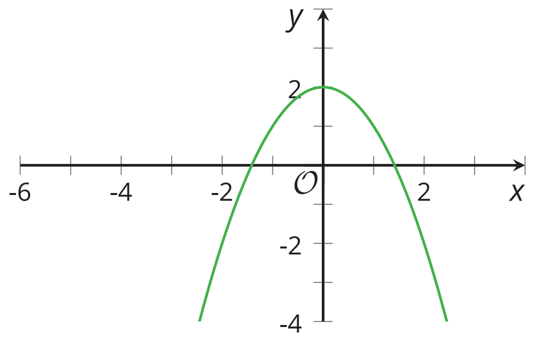
Graph A



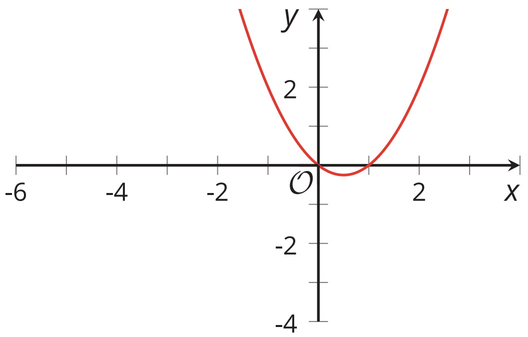
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

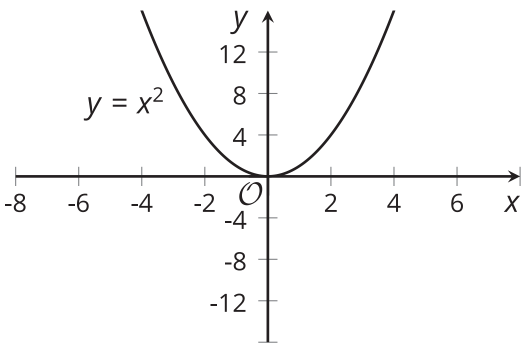
1. $y = x^2 + x$
2. $y = -x^2 + 2$
3. $y = x^2 - x$
4. $y = x^2 + 3x$

2. Complete the table without graphing the equations.

equation	x-intercepts	x-coordinate of the vertex
$y = x^2 + 12x$		
$y = x^2 - 3x$		
$y = -x^2 + 16x$		
$y = -x^2 - 24x$		

3. Here is a graph that represents $y = x^2$.

a. Describe what would happen to the graph if the original equation were changed to $y = x^2 - 6x$. Predict the x- and y-intercepts of the graph and the quadrant where the vertex is located.



b. Sketch the graph of the equation $y = x^2 - 6x$ on the same coordinate plane as $y = x^2$.

4. Select **all** equations whose graph opens upward.

A. $y = -x^2 + 9x$

B. $y = 10x - 5x^2$

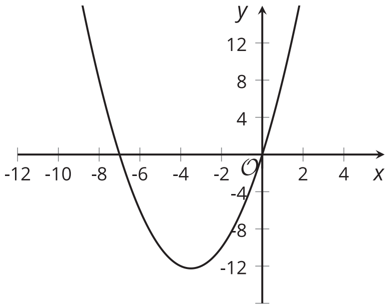
C. $y = (2x - 1)^2$

D. $y = (1 - x)(2 + x)$

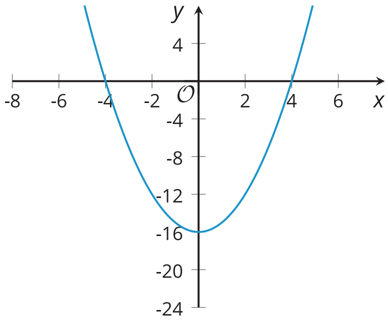
E. $y = x^2 - 8x - 7$

5. *Technology required*. Write an equation for a function that can be represented by each given graph. Then, use graphing technology to check each equation you wrote.

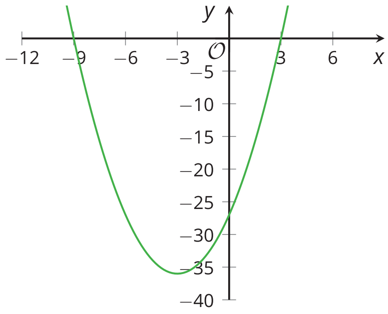
Graph 1



Graph 2



Graph 3



6. Match each quadratic expression that is written as a product with an equivalent expression that is expanded.

A. $(x+3)(x+4)$	1. $x^2+10x+21$
B. $(x+3)(x+7)$	2. $3x^2+13x+12$
C. $(3x+4)(x+3)$	3. $3x^2+22x+7$
D. $(x+7)(3x+1)$	4. $x^2+7x+12$

(From Unit 6, Lesson 8.)

7. When buying a home, many mortgage companies require a down payment of 20% of the price of the house. What is the down payment on a $125,000 home?

(From Unit 5, Lesson 14.)

8. A bank loans $4,000 to a customer at a $9\frac{1}{2}\%$ annual interest rate.

Write an expression to represent how much the customer will owe, in dollars, after 5 years without payment.

(From Unit 5, Lesson 15.)

Lesson 14: Graphs That Represent Situations

14.1: A Jumping Frog

The height in inches of a frog's jump is modeled by the equation $h(t) = 60t - 75t^2$ where the time, t, after it jumped is measured in seconds.

1. Find $h(0)$ and $h(0.8)$. What do these values mean in terms of the frog's jump?

2. How much time after it jumped did the frog reach the maximum height? Explain how you know.

14.2: A Catapulted Pumpkin

The equation $h = 2 + 23.7t - 4.9t^2$ represents the height of a pumpkin that is catapulted up in the air as a function of time, t, in seconds. The height is measured in meters above ground. The pumpkin is shot up at a vertical velocity of 23.7 meters per second.

1. Without writing anything down, consider these questions:
    - What do you think the 2 in the equation tells us in this situation? What about the $-4.9t^2$?
    - If we graph the equation, will the graph open upward or downward? Why?
    - Where do you think the vertical intercept would be?
    - What about the horizontal intercepts?
2. Graph the equation using graphing technology.

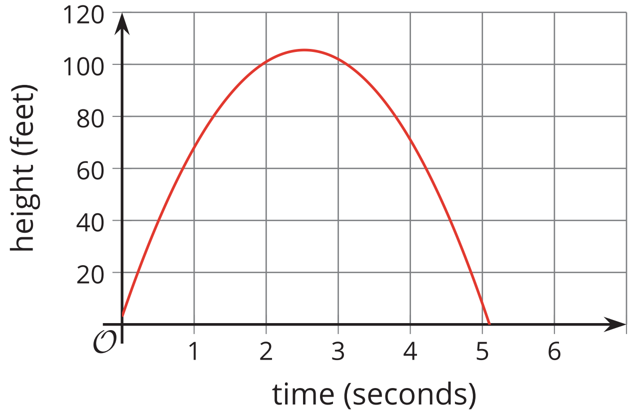
3. Identify the vertical and horizontal intercepts, and the vertex of the graph. Explain what each point means in this situation.

Are you ready for more?

What approximate vertical velocity would this pumpkin need for it stay in the air for about 10 seconds? (Assume that it is still shot from 2 feet in the air and that the effect of gravity pulling it down is the same.)

14.3: Flight of Two Baseballs

Here is a graph that represents the height of a baseball, h, in feet as a function of time, t, in seconds after it was hit by Player A.



The function g defined by $g(t) = (\text{-}16t - 1)(t - 4)$ also represents the height in feet of a baseball t seconds after it was hit by Player B. Without graphing function g, answer the following questions and explain or show how you know.

1. Which player's baseball stayed in flight longer?

2. Which player's baseball reached a greater maximum height?

3. How can you find the height at which each baseball was hit?

14.4: Info Gap: Rocket Math

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

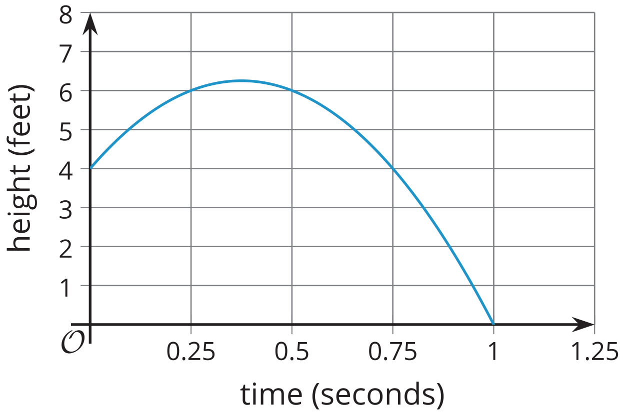
If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Lesson 14 Summary

Let's say a tennis ball is hit straight up in the air, and its height in feet above the ground is modeled by the equation $f(t) = 4 + 12t - 16t^2$. Here is a graph that represents the function, from the time the tennis ball was hit until the time it reached the ground.



In the graph, we can see some information we already know, and some new information:

- The 4 in the equation means the graph of the function intersects the vertical axis at 4. It shows that the tennis ball was 4 feet off the ground at $t = 0$, when it was hit.
- The horizontal intercept is $(1, 0)$. It tells us that the tennis ball hits the ground 1 second after it was hit.
- The vertex of the graph is at approximately $(0.4, 6.3)$. This means that about 0.4 second after the ball was hit, it reached the maximum height of about 6.3 feet.

The equation can be written in factored form as $f(t) = (\text{-}16t - 4)(t - 1)$. From this form, we can see that the zeros of the function are $t = 1$ and $t = \text{-}\frac{1}{4}$. The negative zero, $\text{-}\frac{1}{4}$, is not meaningful in this situation, because the time before the ball was hit is irrelevant.

Lesson 14: Graphs That Represent Situations

Cool Down: Beach Ball Trajectory

The equation $y = (\text{-}16t - 2)(t - 1)$ represents the height in feet of a beach ball thrown by a child as a function of time, t, in seconds.

1. Find the zeros of the function. Explain or show your reasoning.

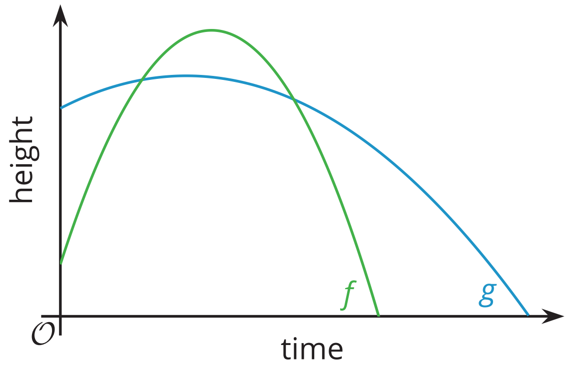
2. What do the zeros tell us in this situation? Are both zeros meaningful?

3. From what height is the beach ball thrown? Explain or show your reasoning.

Unit 6 Lesson 14 Cumulative Practice Problems

1. Here are graphs of functions f and g.

   Each represents the height of an object being launched into the air as a function of time.



   a. Which object was launched from a higher point?

   b. Which object reached a higher point?

   c. Which object was launched with the higher upward velocity?

   d. Which object landed last?

2. *Technology required*. The function h given by $h(t) = (1 - t)(8 + 16t)$ models the height of a ball in feet, t seconds after it was thrown.

   a. Find the zeros of the function. Show or explain your reasoning.

   b. What do the zeros tell us in this situation? Are both zeros meaningful?

   c. From what height is the ball thrown? Explain your reasoning.

   d. About when does the ball reach its highest point, and about how high does the ball go? Show or explain your reasoning.

3. The height in feet of a thrown football is modeled by the equation $f(t) = 6 + 30t - 16t^2$, where time t is measured in seconds.

   a. What does the constant 6 mean in this situation?

   b. What does the $30t$ mean in this situation?

   c. How do you think the squared term $-16t^2$ affects the value of the function f? What does this term reveal about the situation?

4. The height in feet of an arrow is modeled by the equation $h(t) = (1 + 2t)(18 - 8t)$, where t is seconds after the arrow is shot.

   a. When does the arrow hit the ground? Explain or show your reasoning.

   b. From what height is the arrow shot? Explain or show your reasoning.

5. Two objects are launched into the air.

   - The height, in feet, of Object A is given by the equation $f(t) = 4 + 32t - 16t^2$.
   - The height, in feet, of the Object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$. In both functions, t is seconds after launch.

   a. Which object was launched from a greater height? Explain how you know.

   b. Which object was launched with a greater upward velocity? Explain how you know.

6. a. Predict the x- and y-intercepts of the graph of the quadratic function defined by the expression $(x + 6)(x - 6)$. Explain how you made your predictions.

   b. *Technology required*. Check your predictions by graphing $y = (x + 6)(x - 6)$.

(From Unit 6, Lesson 10.)

7. *Technology required*. A student needs to get a loan of \$12,000 for the first year of college. Bank A has an annual interest rate of 5.75%, Bank B has an annual interest rate of 7.81%, and Bank C has an annual rate of 4.45%.

   a. If we graph the amount owed for each loan as a function of years without payment, predict what the three graphs would look like. Describe or sketch your prediction.

   b. Use graphing technology to plot the graph of each loan balance.

   c. Based on your graph, how much would the student owe for each loan when they graduate from college in four years?

   d. Based on your graph, if no payments are made, how much would the student owe for each loan after 10 years?

   (From Unit 5, Lesson 15.)

8. *Technology required*. The functions f and g are given by $f(x) = 13x + 6$ and $g(x) = 0.1 \cdot (1.4)^x$.

   a. Which function eventually grows faster, f or g? Explain how you know.

   b. Use graphing technology to decide when the graphs of f and g meet.

   (From Unit 5, Lesson 19.)

Lesson 15: Vertex Form

15.1: Notice and Wonder: Two Sets of Equations

What do you notice? What do you wonder?

Set 1:

$$f(x) = x^2 + 4x$$

$$g(x) = x(x + 4)$$

$$h(x) = (x + 2)^2 - 4$$

Set 2:

$$p(x) = \text{-}x^2 + 6x - 5$$

$$q(x) = (5 - x)(x - 1)$$

$$r(x) = \text{-}1(x - 3)^2 + 4$$

15.2: A Whole New Form

Here are two sets of equations for quadratic functions you saw earlier. In each set, the expressions that define the output are equivalent.

Set 1:

$$f(x) = x^2 + 4x$$

$$g(x) = x(x + 4)$$

$$h(x) = (x + 2)^2 - 4$$

Set 2:

$$p(x) = \text{-}x^2 + 6x - 5$$

$$q(x) = (5 - x)(x - 1)$$

$$r(x) = \text{-}1(x - 3)^2 + 4$$

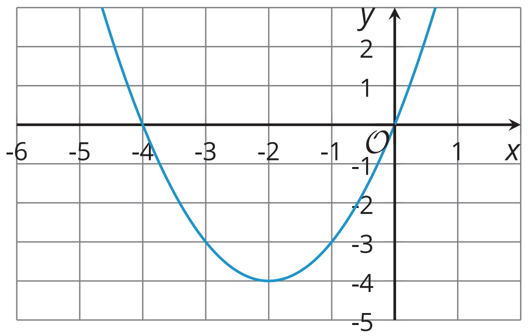
The expression that defines h is written in **vertex form**. We can show that it is equivalent to the expression defining f by expanding the expression:

$$\begin{aligned}(x + 2)^2 - 4 &= (x + 2)(x + 2) - 4\\ &= x^2 + 2x + 2x + 4 - 4\\ &= x^2 + 4x\end{aligned}$$

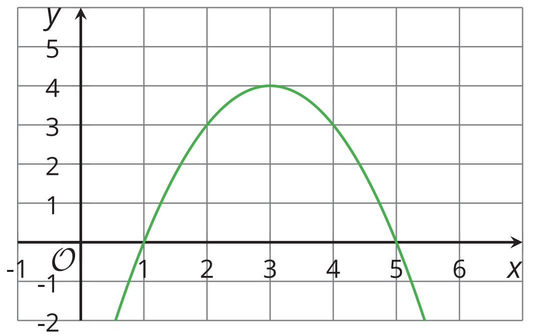
1. Show that the expressions defining r and p are equivalent.

2. Here are graphs representing the quadratic functions. Why do you think expressions such as those defining h and r are said to be written in vertex form?

Graph of h



Graph of r



15.3: Playing with Parameters

1. Using graphing technology, graph $y = x^2$. Then, add different numbers to x before it is squared (for example, $y = (x + 4)^2$, $y = (x - 3)^2$) and observe how the graph changes. Record your observations.

2. Graph $y = (x - 1)^2$. Then, experiment with each of the following changes to the function and see how they affect the graph and the vertex:

    a. Adding different constant terms to $(x - 1)^2$ (for example: $(x - 1)^2 + 5$, $(x - 1)^2 - 9$).

    b. Multiplying $(x - 1)^2$ by different coefficients (for example: $y = 3(x - 1)^2$, $y = -2(x - 1)^2$).

3. Without graphing, predict the coordinates of the vertex of the graphs of these quadratic functions, and predict whether the graph opens up or opens down. Ignore the last row until the next question.

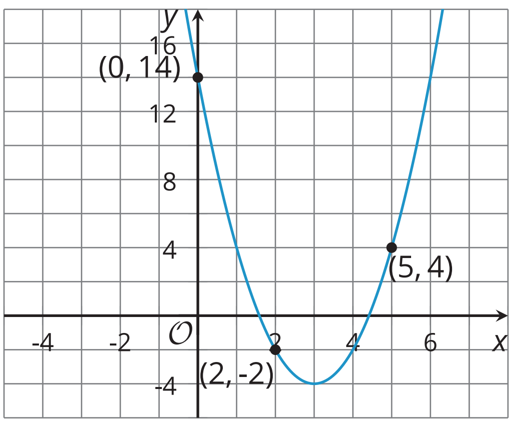
equations	coordinates of vertex	graph opens up or down?
$y = (x + 10)^2$		
$y = (x - 4)^2 + 8$		
$y = \text{-}(x - 4)^2 + 8$		
$y = x^2 - 7$		
$y = \frac{1}{2}(x + 3)^2 - 5$		
$y = \text{-}(x + 100)^2 + 50$		
$y = a(x + m)^2 + n$		

4. Use graphing technology to check your predictions. If they are incorrect, revise them. Then, complete the last row of the table.

Are you ready for more?

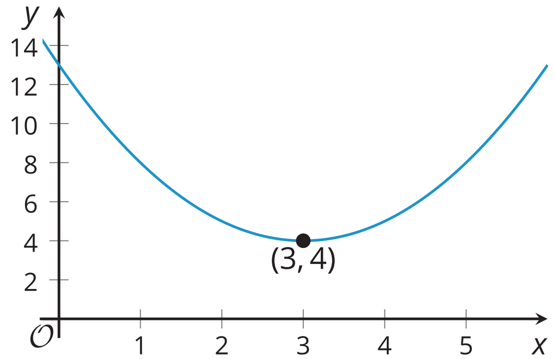
1. What is the vertex of this graph?

2. Find a quadratic equation whose graph has the same vertex and adjust it, if needed, so that it has the graph provided.



Lesson 15 Summary

Sometimes the expressions that define quadratic functions are written in **vertex form**. For example, if the function f is defined by $(x-3)^2+4$, which is in vertex form, we can write $f(x)=(x-3)^2+4$ and draw this graph to represent f.



The vertex form can tell us about the coordinates of the vertex of the graph of a quadratic function. The expression $(x-3)^2$ reveals that the vertex has x-coordinate 3, and the constant term of 4 reveals its y-coordinate. Here the vertex represents the minimum value of the function f, and its graph opens upward.

In general, a quadratic function expressed in vertex form is written as:

$$y=a(x-h)^2+k$$

The vertex of its graph is at (h,k). The graph of the quadratic function opens upward when the coefficient a is positive and opens downward when a is negative.

In future lessons, we will explore further how a, h, and k affect the graph of a quadratic function.

Lesson 15: Vertex Form

Cool Down: Visualizing A Graph

The function f is given by $f(x) = (x + 3)^2 - 1$.

1. Write the coordinates of the vertex of the graph of f.

2. Does the graph open up or down? Explain how you know.

Unit 6 Lesson 15 Cumulative Practice Problems

1. Select **all** of the quadratic expressions in vertex form.

   A. $(x-2)^2+1$

   B. x^2-4

   C. $x(x+1)$

   D. $(x+3)^2$

   E. $(x-4)^2+6$

2. Here are two equations. One defines function m and the other defines function p.

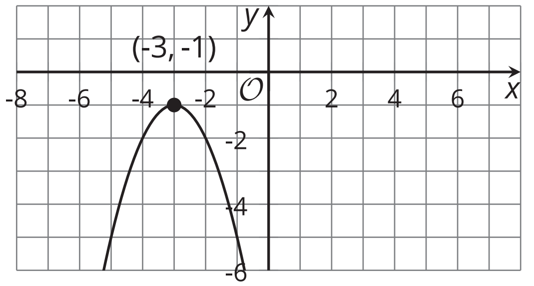
   $m(x)=x(x+6)$ $\qquad$ $p(x)=(x+3)^2-9$

   a. Show that the expressions defining m and p are equivalent.

   b. What is the vertex of the graph of m? Explain how you know.

   c. What are the x-intercepts of the graph of p? Explain how you know.

3. Which equation is represented by the graph?



A. $y = (x-1)^2 + 3$

B. $y = (x-3)^2 + 1$

C. $y = \text{-}(x+3)^2 - 1$

D. $y = \text{-}(x-3)^2 + 1$

4. For each equation, write the coordinates of the vertex of the graph that represents the equation.

a. $y = (x-3)^2 + 5$

b. $y = (x+7)^2 + 3$

c. $y = (x-4)^2$

d. $y = x^2 - 1$

e. $y = 2(x+1)^2 - 5$

f. $y = \text{-}2(x+1)^2 - 5$

5. For each function, write the coordinates of the vertex of its graph and tell whether the graph opens up or down.

function	coordinates of vertex	graph opens up or down?
$f(x) = (x-4)^2 - 5$		
$g(x) = \text{-}x^2 + 5$		
$h(x) = 2(x+1)^2 - 4$		

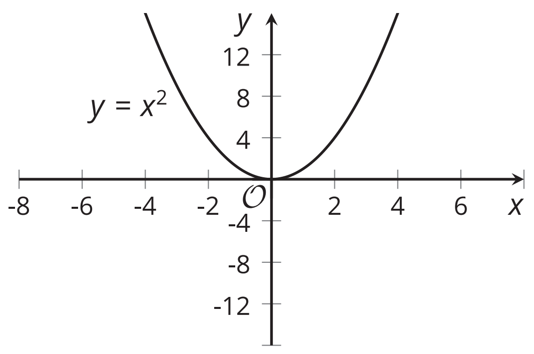
6. Here is a graph that represents $y = x^2$.

   a. Describe what would happen to the graph if the original equation were modified as follows:

      i. $y = -x^2$

      ii. $y = 3x^2$

      iii. $y = x^2 + 6$



   b. Sketch the graph of the equation $y = -3x^2 + 6$ on the same coordinate plane as $y = x^2$.

(From Unit 6, Lesson 12.)

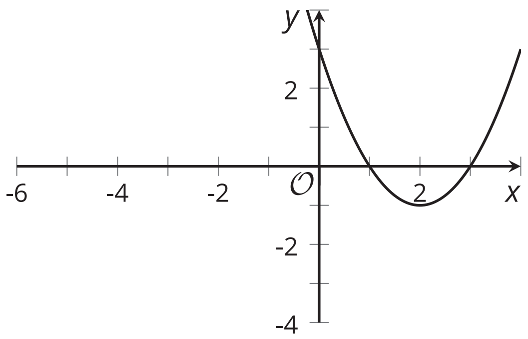
7. Noah is going to put \$2,000 in a savings account. He plans on putting the money in an account and leaving it there for 5 years. He can put the money in an account that pays 1% interest monthly, an account that pays 6% interest every six months, or an account that pays 12% interest annually.

   Which account will give him the most money in his account at the end of the 5 years?

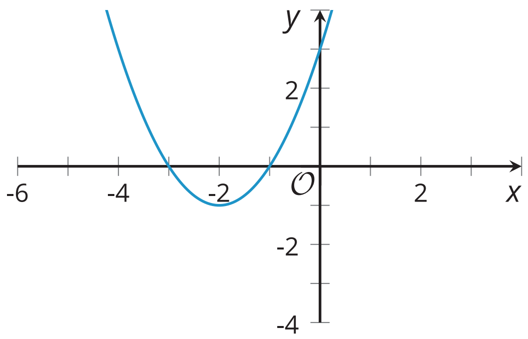
(From Unit 5, Lesson 16.)

8. Here are four graphs. Match each graph with a quadratic equation that it represents.

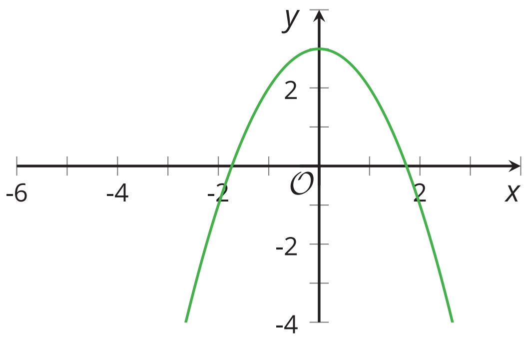
Graph A



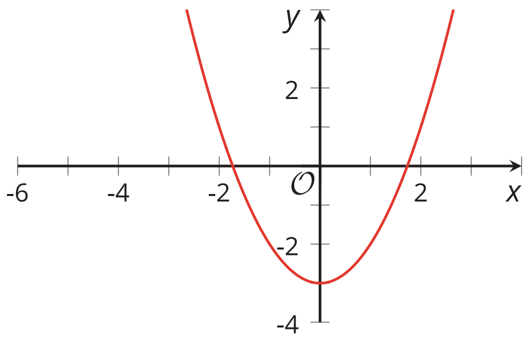
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. $y = -x^2 + 3$
2. $y = (x + 1)(x + 3)$
3. $y = x^2 - 3$
4. $y = (x - 1)(x - 3)$

(From Unit 6, Lesson 12.)

9. The table shows some input and output values of function f. Describe a possible rule for the function by using words or by writing an equation.

x	$f(x)$
-3	-8
0	-2
4	6
10	18

(From Unit 4, Lesson 5.)

Lesson 16: Graphing from the Vertex Form

16.1: Which Form to Use?

Expressions in different forms can be used to define the same function. Here are three ways to define a function f.

$f(x) = x^2 - 4x + 3$	(standard form)
$f(x) = (x-3)(x-1)$	(factored form)
$f(x) = (x-2)^2 - 1$	(vertex form)

Which form would you use if you want to find the following features of the graph of f? Be prepared to explain your reasoning.

1. the x-intercepts
2. the vertex
3. the y-intercept

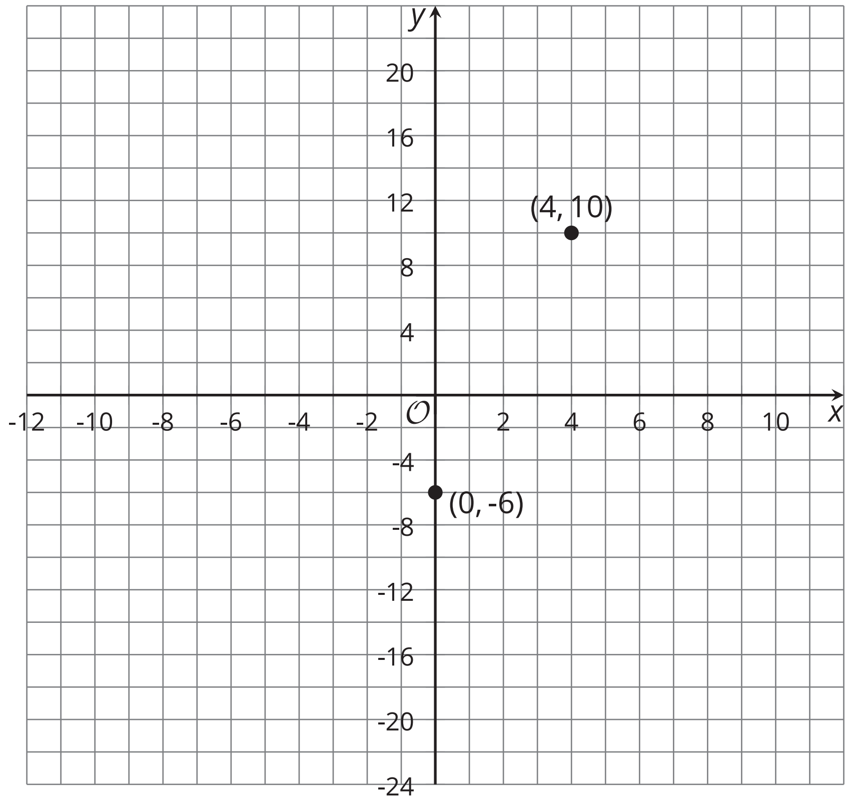
16.2: Sharing a Vertex

Here are two equations that define quadratic functions.

$$p(x) = \text{-}(x-4)^2 + 10$$
$$q(x) = \frac{1}{2}(x-4)^2 + 10$$

1. The graph of p passes through $(0, \text{-}6)$ and $(4, 10)$, as shown on the coordinate plane.

   Find the coordinates of another point on the graph of p. Explain or show your reasoning. Then, use the points to sketch and label the graph.



2. On the same coordinate plane, identify the vertex and two other points that are on the graph of q. Explain or show your reasoning. Sketch and label the graph of q.

3. Priya says, "Once I know the vertex is $(4, 10)$, I can find out, without graphing, whether the vertex is the maximum or the minimum of function p. I would just compare the coordinates of the vertex with the coordinates of a point on either side of it."

   Complete the table and then explain how Priya might have reasoned about whether the vertex is the minimum or maximum.

x	3	4	5
$p(x)$		10	

Are you ready for more?

1. Write a the equation for a quadratic function whose graph has the vertex at $(2, 3)$ and contains the point $(0, \text{-}5)$?

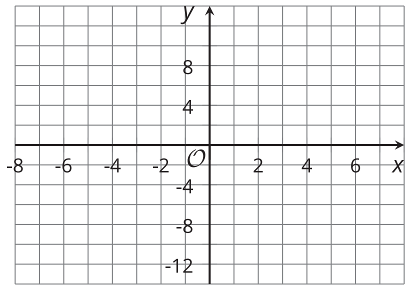
2. Sketch a graph of your function.

16.3: Card Sort: Matching Equations with Graphs

Your teacher will give you a set of cards. Each card contains an equation or a graph that represents a quadratic function. Take turns matching each equation to a graph that represents the same function.

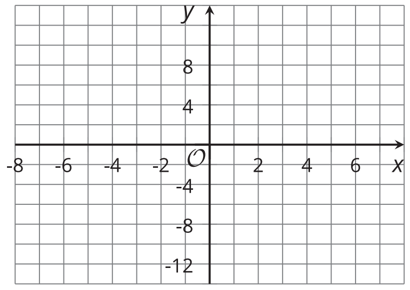
- For each pair of cards that you match, explain to your partner how you know they belong together.
- For each pair of cards that your partner matches, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
- Once all the cards are matched, record the equation, the label and a sketch of the corresponding graph, and write a brief note or explanation about how you knew they were a match.

Equation:



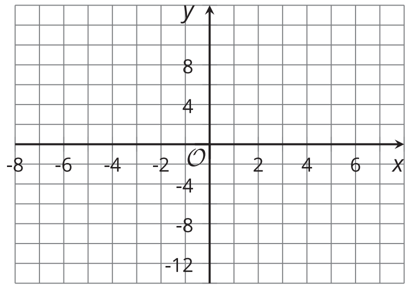
Explanation:

Equation:



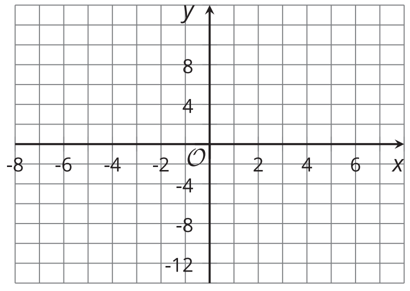
Explanation:

Equation:



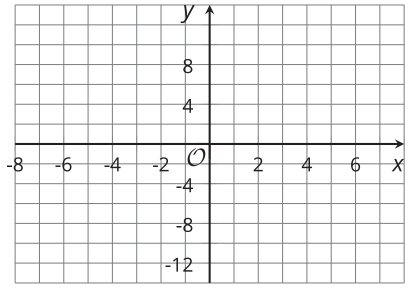
Explanation:

Equation:



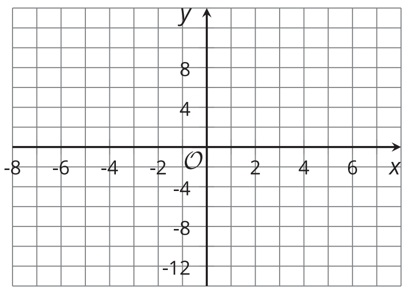
Explanation:

Equation:



Explanation:

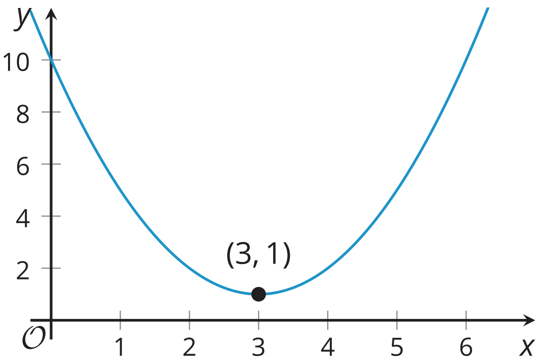
Equation:



Explanation:

Lesson 16 Summary

We saw that vertex form is especially helpful for finding the vertex of a graph of a quadratic function. For example, we can tell that the function p given by $p(x) = (x-3)^2 + 1$ has a vertex at $(3, 1)$.



We also noticed that, when the squared expression $(x-3)^2$ has a positive coefficient, the graph opens upward. This means that the vertex $(3, 1)$ represents the minimum function value.

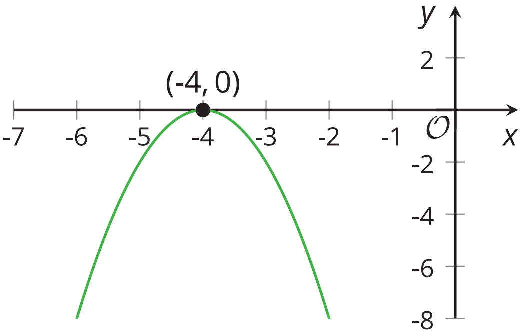
But why does the function p take on its minimum value when x is 3?

Here is one way to explain it: When $x = 3$, the squared term $(x-3)^2$ equals 0, as $(3-3)^2 = 0^2 = 0$. When x is any other value besides 3, the squared term $(x-3)^2$ is a positive number greater than 0. (Squaring any number results in a positive number.) This means that the output when $x \neq 3$ will always be greater than the output when $x = 3$, so the function p has a minimum value at $x = 3$.

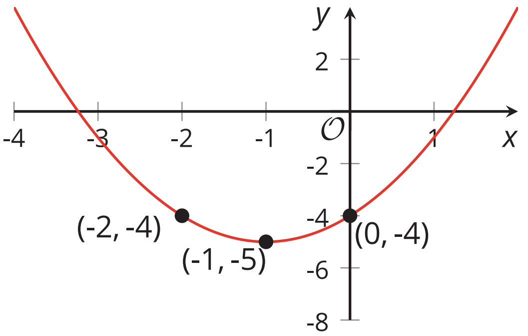
This table shows some values of the function for some values of x. Notice that the output is the least when $x = 3$ and it increases both as x increases and as it decreases.

x	0	1	2	3	4	5	6
$(x-3)^2 + 1$	10	5	2	1	2	5	10

The squared term sometimes has a negative coefficient, for instance: $h(x) = -2(x+4)^2$. The x value that makes $(x+4)^2$ equal 0 is -4, because $(-4+4)^2 = 0^2 = 0$. Any other x value makes $(x+4)^2$ greater than 0. But when $(x+4)^2$ is multiplied by a negative number (-2), the resulting expression, $-2(x+4)^2$, ends up being negative. This means that the output when $x \neq -4$ will always be less than the output when $x = -4$, so the function h has its maximum value when $x = -4$.



Remember that we can find the y-intercept of the graph representing any function we have seen. The y-coordinate of the y-intercept is the value of the function when $x = 0$. If g is defined by $g(x) = (x+1)^2 - 5$, then the y-intercept is $(0, -4)$ because $g(0) = (0+1)^2 - 5 = -4$. Its vertex is at $(-1, -5)$. Another point on the graph with the same y-coordinate is located the same horizontal distance from the vertex but on the other side.



Matching Equations with Graphs

$$f(x) = (x-1)^2 + 4$$

Matching Equations with Graphs

$$g(x) = \text{-}(x-4)^2 + 1$$

Matching Equations with Graphs

$$h(x) = (x+1)^2 - 4$$

Matching Equations with Graphs

$$p(x) = \text{-}(x+1)^2 - 4$$

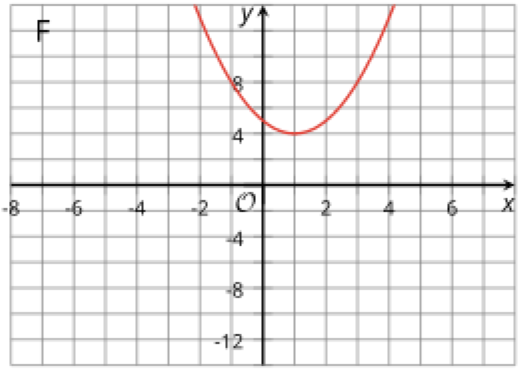
Matching Equations with Graphs

$$q(x) = 2(x-4)^2 + 1$$

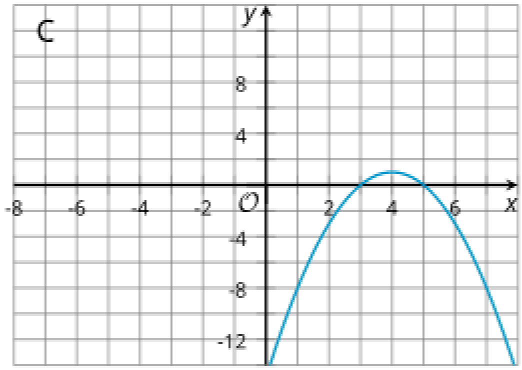
Matching Equations with Graphs

$$r(x) = (x+4)^2 - 1$$

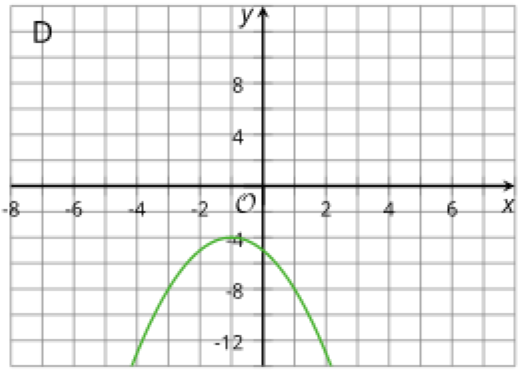
Matching Equations with Graphs



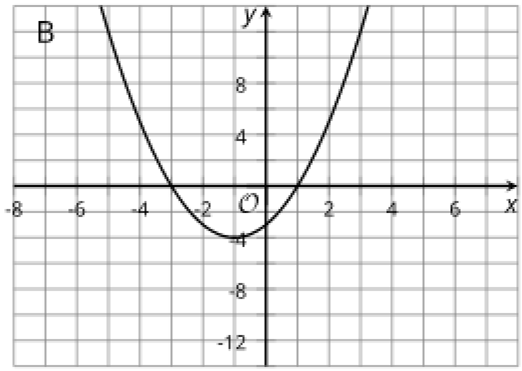
Matching Equations with Graphs



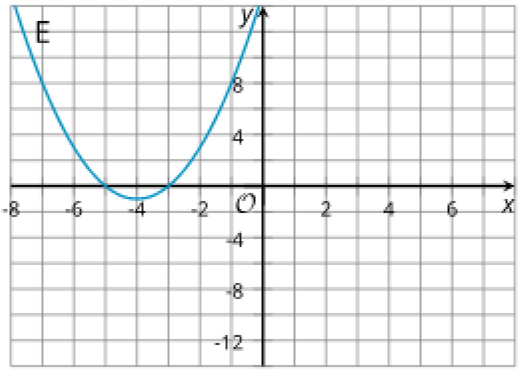
Matching Equations with Graphs



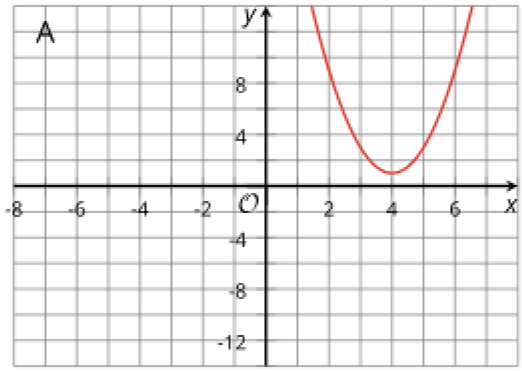
Matching Equations with Graphs



Matching Equations with Graphs



Matching Equations with Graphs



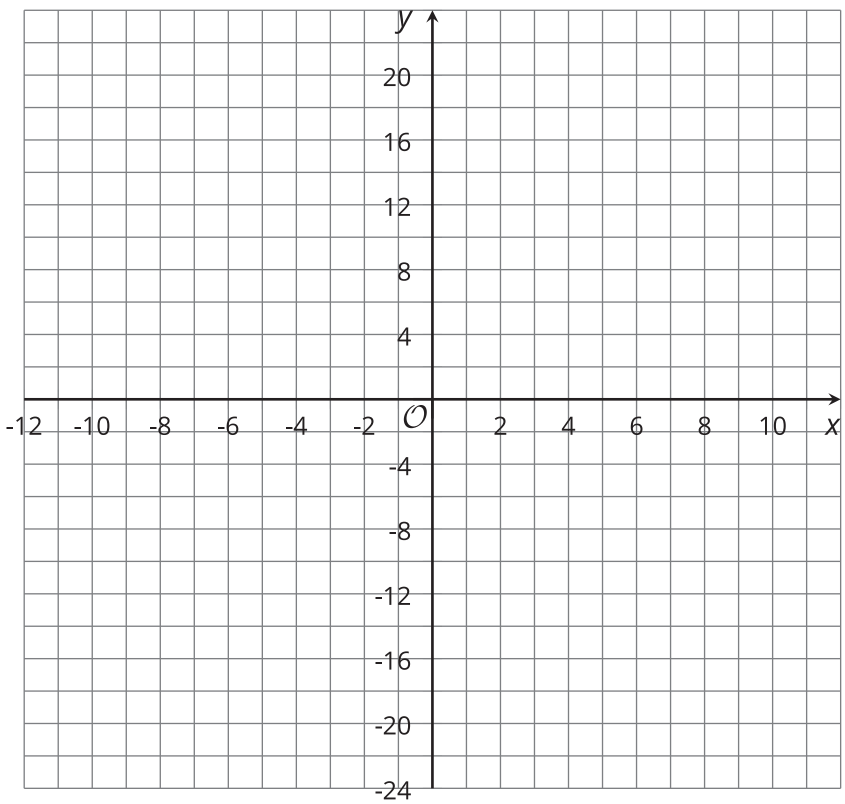
Lesson 16: Graphing from the Vertex Form

Cool Down: Sketching A Graph

1. What are the coordinates of the vertex of the graph defined by $y = (x - 3)^2 + 2$?

2. Find the coordinates of two other points on the graph. Show your reasoning.

3. Sketch a graph that represents the equation.



Unit 6 Lesson 16 Cumulative Practice Problems

1. Which equation can be represented by a graph with a vertex at $(1, 3)$?

   A. $y = (x-1)^2 + 3$

   B. $y = (x+1)^2 + 3$

   C. $y = (x-3)^2 + 1$

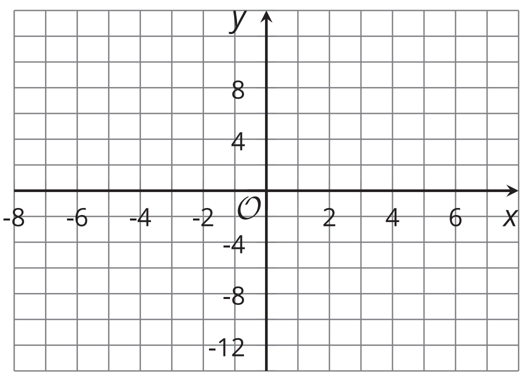
   D. $y = (x+3)^2 + 1$

2. a. Where is the vertex of the graph that represents $y = (x-2)^2 - 8$?

   b. Where is the y-intercept? Explain how you know.

   c. Identify one other point on the graph of the equation. Explain or show how you know.

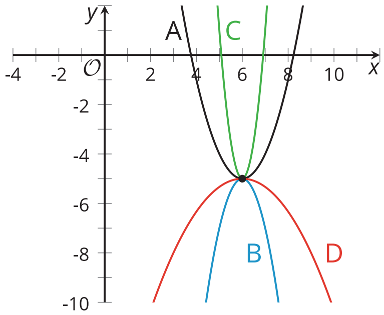
   d. Sketch a graph that represents the equation.



3. The function v is defined by $v(x) = \frac{1}{2}(x+5)^2 - 7$.

   Without graphing, determine if the vertex of the graph representing v shows the minimum or maximum value of the function. Explain how you know.

4. Match each graph to an equation that represents it.



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. $y = -2(x-6)^2 - 5$

2. $y = (x-6)^2 - 5$

3. $y = 6(x-6)^2 - 5$

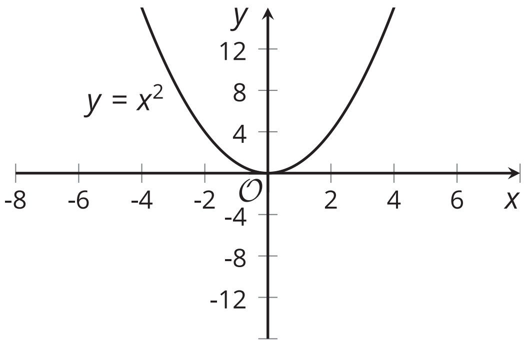
4. $y = -\frac{1}{3}(x-6)^2 - 5$

5. Here is a graph that represents $y = x^2$.

   a. Describe what would happen to the graph if the original equation was changed to:

      i. $y = \frac{1}{2}x^2$

      ii. $y = x^2 - 8$

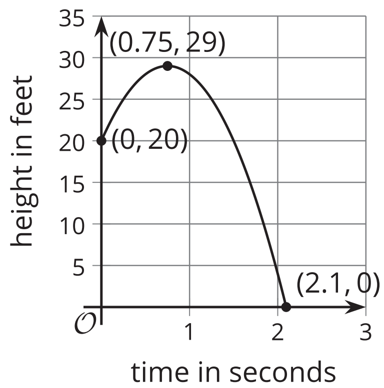


   b. Graph the equation $y = \frac{1}{2}x^2 - 8$ on the same coordinate plane as $y = x^2$.

(From Unit 6, Lesson 12.)

6. Clare throws a rock into the lake. The graph shows the rock's height above the water, in feet, as a function of time in seconds.

   Select **all** the statements that describe this situation.



   A. The vertex of the graph is $(0.75, 29)$.

   B. The y-intercept of the graph is $(2.1, 0)$.

   C. Clare just dropped the rock into the lake.

   D. The maximum height of the rock is about 20 feet.

   E. The rock hits the surface of the water after about 2.1 seconds.

   F. Clare tossed the rock up into the air from a point 20 feet above the water.

(From Unit 6, Lesson 14.)

7. *Technology required.* Two objects are launched into the air.

   - The height, in feet, of Object A is given by the equation $f(t) = 4 + 32t - 16t^2$.
   - The height, in feet, of the Object B is given by the equation $g(t) = 2.5 + 40t - 16t^2$. In both functions, t is seconds after launch.

   Use technology to graph each function in the same graphing window.

   a. What is the maximum height of each object?

   b. Which object hits the ground first? Explain how you know.

(From Unit 6, Lesson 14.)

8. Andre thinks the vertex of the graph of the equation $y = (x + 2)^2 - 3$ is $(2, \text{-}3)$. Lin thinks the vertex is $(\text{-}2, 3)$. Do you agree with either of them?

(From Unit 6, Lesson 15.)

9. The expression $2{,}000 \cdot \left(1.015^{12}\right)^5$ represents the balance, in dollars, in a savings account.

    a. What is the rate of interest paid on the account?

    b. How many years has the account been accruing interest?

    c. How much money was invested?

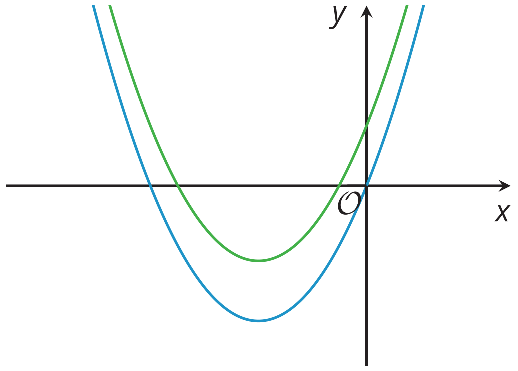
    d. How much money is in the account now?

    e. Write an equivalent expression to represent the balance in the savings account.

(From Unit 5, Lesson 17.)

Lesson 17: Changing the Vertex

17.1: Graphs of Two Functions



Here are graphs representing the functions f and g, given by $f(x) = x(x + 6)$ and $g(x) = x(x + 6) + 4$.

1. Which graph represents each function? Explain how you know.

2. Where does the graph of f meet the x-axis? Explain how you know.

17.2: Shifting the Graph

1. How would you change the equation $y = x^2$ so that the vertex of the graph of the new equation is located at the following coordinates and the graph opens as described?
   a. $(0, 11)$, opens up

   b. $(7, 11)$, opens up

   c. $(7, \text{-}3)$, opens down

2. Use graphing technology to verify your predictions. Adjust your equations if necessary.

3. Kiran graphed the equation $y = x^2 + 1$ and noticed that the vertex is at $(0, 1)$. He changed the equation to $y = (x - 3)^2 + 1$ and saw that the graph shifted 3 units to the right and the vertex is now at $(3, 1)$.

   Next, he graphed the equation $y = x^2 + 2x + 1$, observed that the vertex is at $(\text{-}1, 0)$. Kiran thought, "If I change the squared term x^2 to $(x - 5)^2$, the graph will move 5 units to the right and the vertex will be at $(4, 0)$."

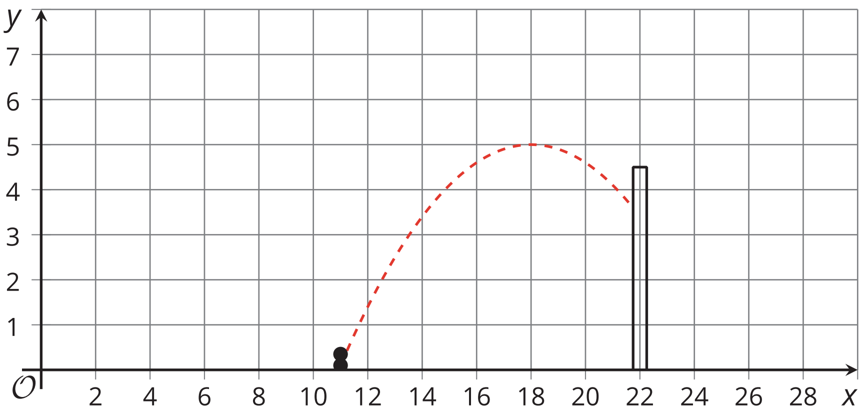
   Do you agree with Kiran? Explain or show your reasoning.

17.3: A Peanut Jumping over a Wall

Mai is learning to create computer animation by programming. In one part of her animation, she uses a quadratic function to model the path of the main character, an animated peanut, jumping over a wall.

Mai uses the equation $y = -0.1(x-h)^2 + k$ to represent the path of the jump. y represents the height of the peanut as a function of the horizontal distance it travels, x.

On the screen, the base of the wall is located at $(22, 0)$, with the top of the wall at $(22, 4.5)$. The dashed curve in the picture shows the graph of 1 equation Mai tried, where the peanut fails to make it over the wall.



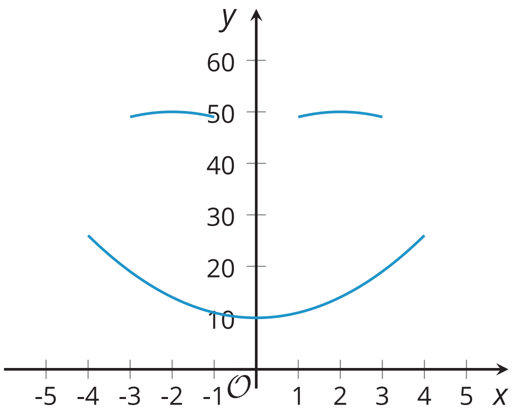
1. What are the values of h and k in this equation?

2. Starting with Mai's equation, choose values for h and k that will guarantee the peanut stays on the screen but also makes it over the wall. Be prepared to explain your reasoning.

17.4: Smiley Face

Do you see 2 “eyes” and a smiling “mouth” on the graph? The 3 arcs on the graph all represent quadratic functions that were initially defined by $y = x^2$, but whose equations were later modified.

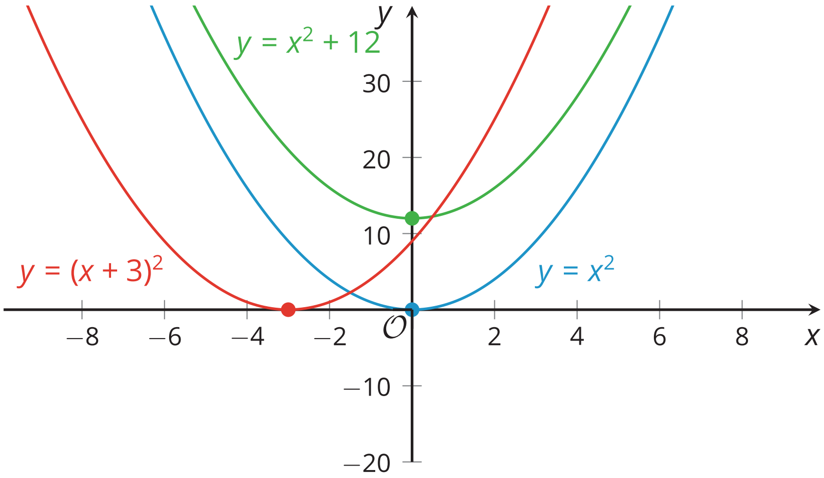
1. Write equations to represent each curve in the smiley face.



2. What domain is used for each function to create this graph?

Lesson 17 Summary

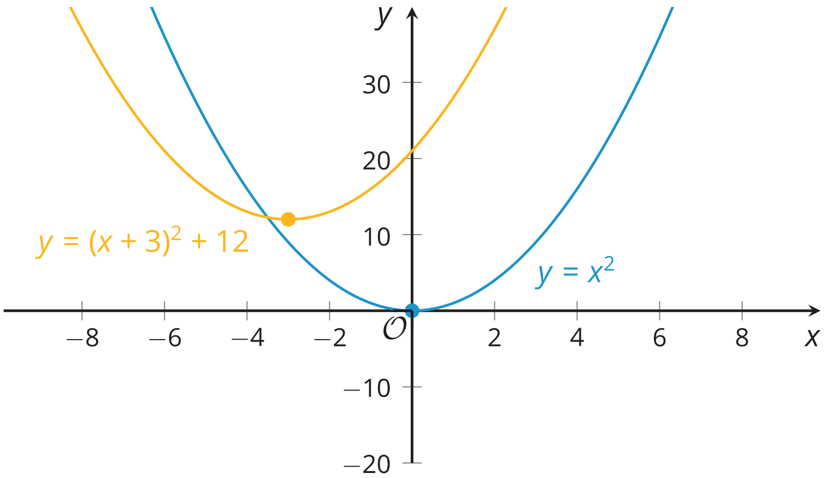
The graphs of $y = x^2$, $y = x^2 + 12$ and $y = (x+3)^2$ all have the same shape but their locations are different. The graph that represents $y = x^2$ has its vertex at $(0, 0)$.



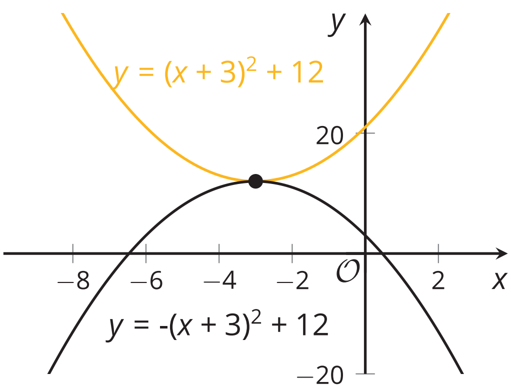
Notice that adding 12 to x^2 raises the graph by 12 units, so the vertex of that graph is at $(0, 12)$. Replacing x^2 with $(x+3)^2$ shifts the graph 3 units to the left, so the vertex is now at $(-3, 0)$.

We can also shift a graph both horizontally and vertically.

The graph that represents $y = (x+3)^2 + 12$ will look like that for $y = x^2$ but it will be shifted 12 units up and 3 units to the left. Its vertex is at $(-3, 12)$.



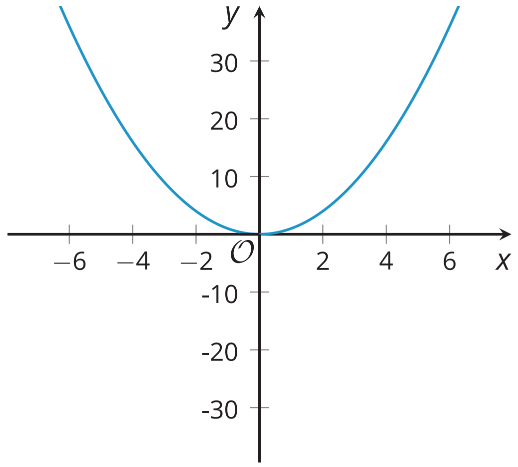
The graph representing the equation $y = -(x+3)^2 + 12$ has the same vertex at $(-3, 12)$, but because the squared term $(x+3)^2$ is multiplied by a negative number, the graph is flipped over horizontally, so that it opens downward.



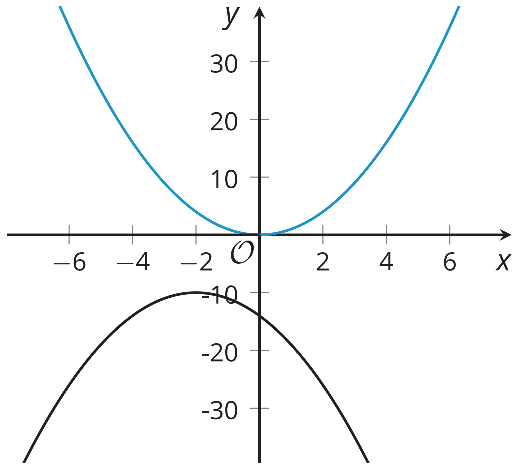
Lesson 17: Changing the Vertex

Cool Down: Nudging A Graph

1. Here is a graph that represents $y = x^2$. On the same coordinate plane, sketch a graph that represents $y = (x - 6)^2 + 10$.

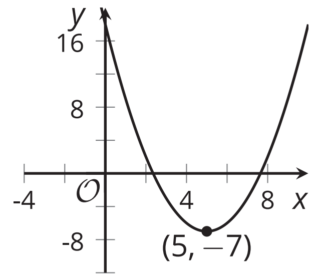


2. The graph representing $y = x^2$ is shifted 2 units to the left, 10 units down, and flipped so that it opens downward, as shown. Write an equation that defines this curve.



Unit 6 Lesson 17 Cumulative Practice Problems

1. Here the graph of quadratic function f.

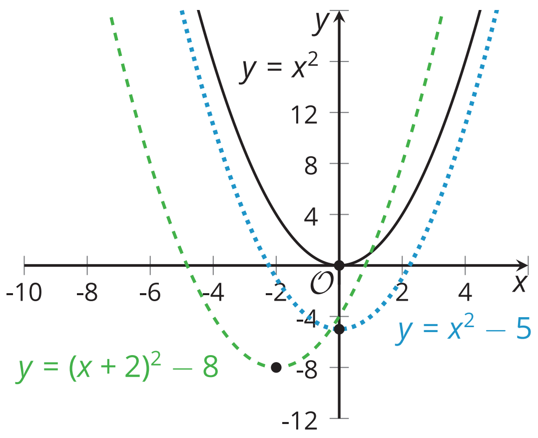
   Andre uses the expression $(x-5)^2+7$ to define f.

   Noah uses the expression $(x+5)^2-7$ to define f.

   Do you agree with either of them? Explain your reasoning.

2. Here are the graphs of $y = x^2$, $y = x^2 - 5$, and $y = (x + 2)^2 - 8$.

   a. How do the 3 graphs compare?



   b. How does the -5 in $x^2 - 5$ affect the graph?

   c. How does the +2 and the -8 in $(x + 2)^2 - 8$ affect the graph?

3. Which equation represents the graph of $y = x^2 + 2x - 3$ moved 3 units to the left?

   A. $y = x^2 + 2x - 6$

   B. $y = (x + 3)^2 + 2x - 3$

   C. $y = (x + 3)^2 + 2(x + 3)$

   D. $y = (x + 3)^2 + 2(x + 3) - 3$

4. Select **all** the equations with a graph whose vertex has *both* a positive x- and a positive y-coordinate.

A. $y = x^2$

B. $y = (x - 1)^2$

C. $y = (x - 3)^2 + 2$

D. $y = 2(x - 4)^2 - 5$

E. $y = 0.5(x + 2)^2 + 6$

F. $y = \text{-}(x - 4)^2 + 3$

G. $y = \text{-}2(x - 3)^2 + 1$

5. The height in feet of a soccer ball is modeled by the equation $g(t) = 2 + 50t - 16t^2$, where time t is measured in seconds after it was kicked.

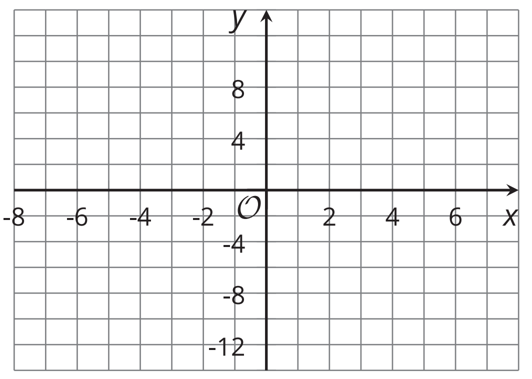
   a. How far above the ground was the ball when kicked?

   b. What was the initial upward velocity of the ball?

   c. Why is the coefficient of the squared term negative?

(From Unit 6, Lesson 14.)

6. a. What is the vertex of the graph of the function f defined by $f(x) = -(x-3)^2 + 6$?



b. Identify the y-intercept and one other point on of the graph of this function.

c. Sketch the graph of f.

(From Unit 6, Lesson 16.)

7. At 6:00 a.m., Lin began hiking. At noon, she had hiked 12 miles. At 4:00 p.m., Lin finished hiking with a total trip of 26 miles.

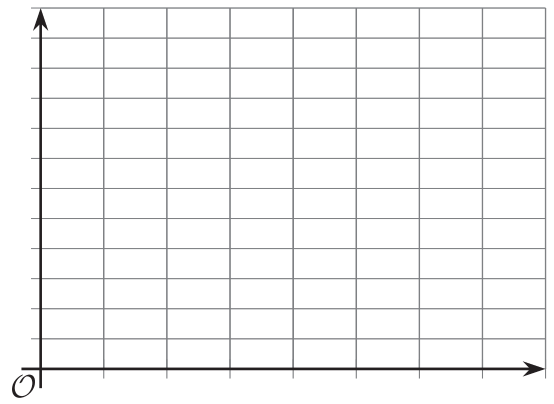
During which time interval was Lin hiking faster? Explain how you know.

(From Unit 4, Lesson 7.)

8. Kiran bought a smoothie every day for a week. Smoothies cost \$3 each. The amount of money he spends, in dollars, is a function of the number of days of buying smoothies.

    a. Sketch a graph of this function. Be sure to label the axes.

    b. Describe the domain and range of this function.



(From Unit 4, Lesson 11.)

9. A deposit of \$500 has been made in an interest-bearing account. No withdrawals or other deposits (aside from earned interest) are made for 5 years.

    Write an expression to represent the account balance for each of the following situations.

    a. 6.5% interest calculated monthly

    b. 6.5% interest calculated every two months

    c. 6.5% interest calculated quarterly

    d. 6.5% interest calculated semi-annually

(From Unit 5, Lesson 18.)

10. *Technology required*. Function h is defined by $h(x) = 5x + 7$ and function k is defined by $k(x) = (1.005)^x$.

    a. Complete the table with values of $h(x)$ and $k(x)$. When necessary, round to 2 decimal places.

x	$h(x)$	$k(x)$
1		
10		
50		
100		

    b. Which function do you think *eventually* grows faster? Explain your reasoning.

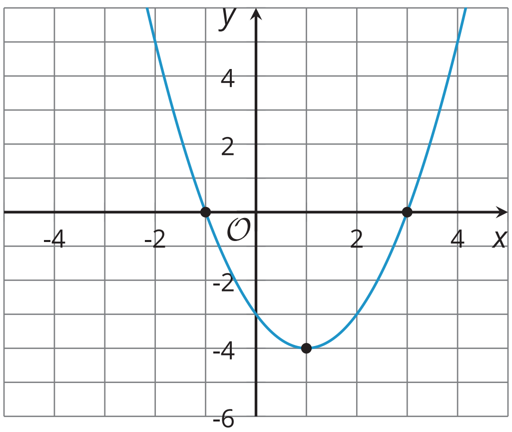
    c. Use graphing technology to verify your answer to the previous question.

    (From Unit 5, Lesson 19.)

Lesson 11 Summary

The function f given by $f(x) = (x+1)(x-3)$ is written in factored form. Recall that this form is helpful for finding the zeros of the function (where the function has the value 0) and telling us the x-intercepts on the graph representing the function.

Here is a graph representing f. It shows 2 x-intercepts at $x = \text{-}1$ and $x = 3$.



If we use -1 and 3 as inputs to f, what are the outputs?

- $f(\text{-}1) = (\text{-}1+1)(\text{-}1-3) = (0)(\text{-}4) = 0$
- $f(3) = (3+1)(3-3) = (4)(0) = 0$

Because the inputs -1 and 3 produce an output of 0, they are the zeros of the function f. And because both x values have 0 for their y value, they also give us the x-intercepts of the graph (the points where the graph crosses the x-axis, which always have a y-coordinate of 0). So, the zeros of a function have the same values as the x-coordinates of the x-intercepts of the graph of the function.

The factored form can also help us identify the vertex of the graph, which is the point where the function reaches its minimum value. Notice that the x-coordinate of the vertex is 1, and that 1 is halfway between -1 and 3. Once we know the x-coordinate of the vertex, we can find the y-coordinate by evaluating the function: $f(1) = (1+1)(1-3) = 2(\text{-}2) = \text{-}4$. So the vertex is at $(1, \text{-}4)$.

When a quadratic function is in standard form, the y-intercept is clear: its y-coordinate is the constant term c in $ax^2 + bx + c$. To find the y-intercept from factored form, we can evaluate the function at $x = 0$, because the y-intercept is the point where the graph has an input value of 0. $f(0) = (0+1)(0-3) = (1)(\text{-}3) = \text{-}3$.