NYS
Algebra II
Regents Exams

June 2016

June 2017

June 2018

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High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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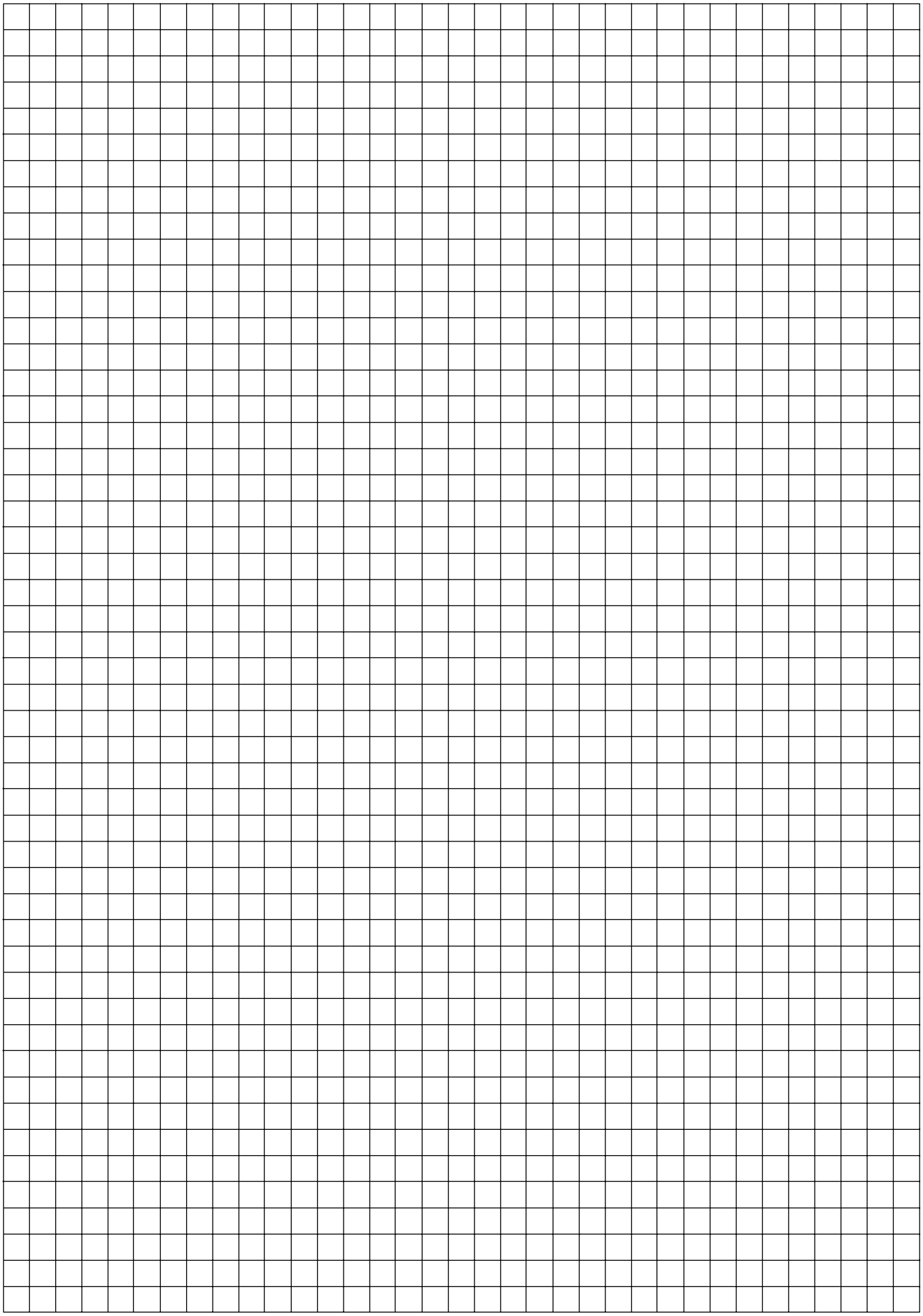

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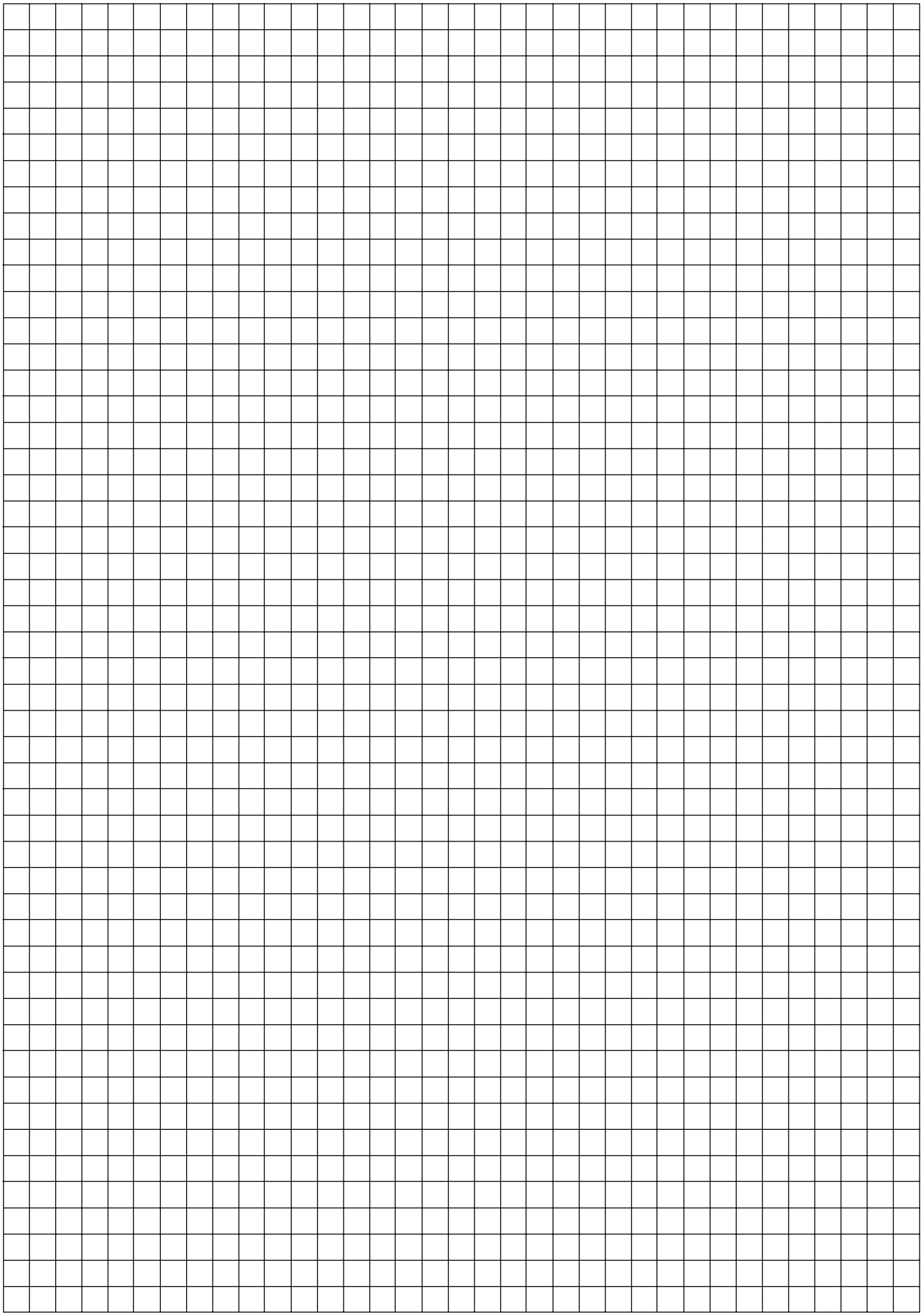

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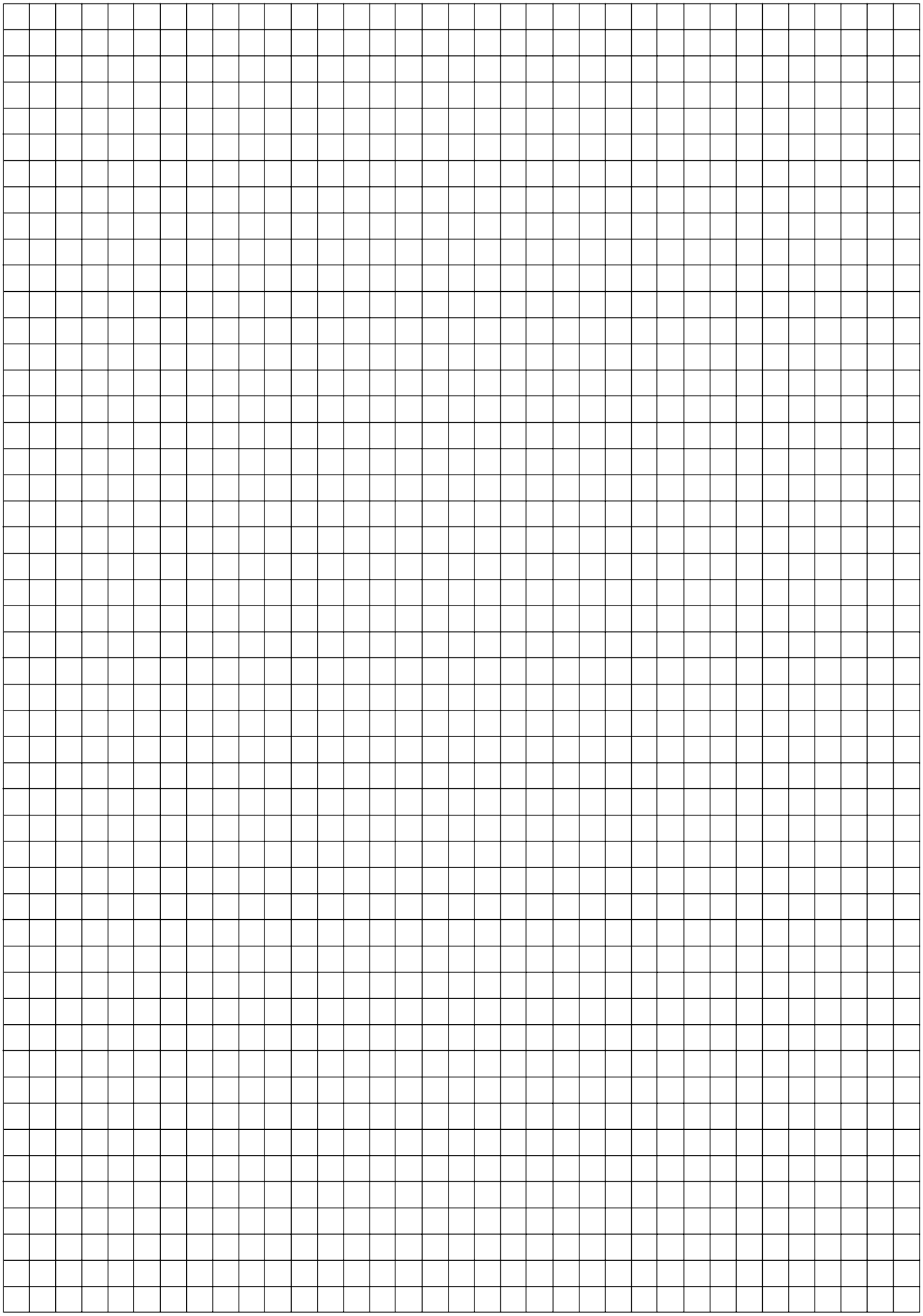

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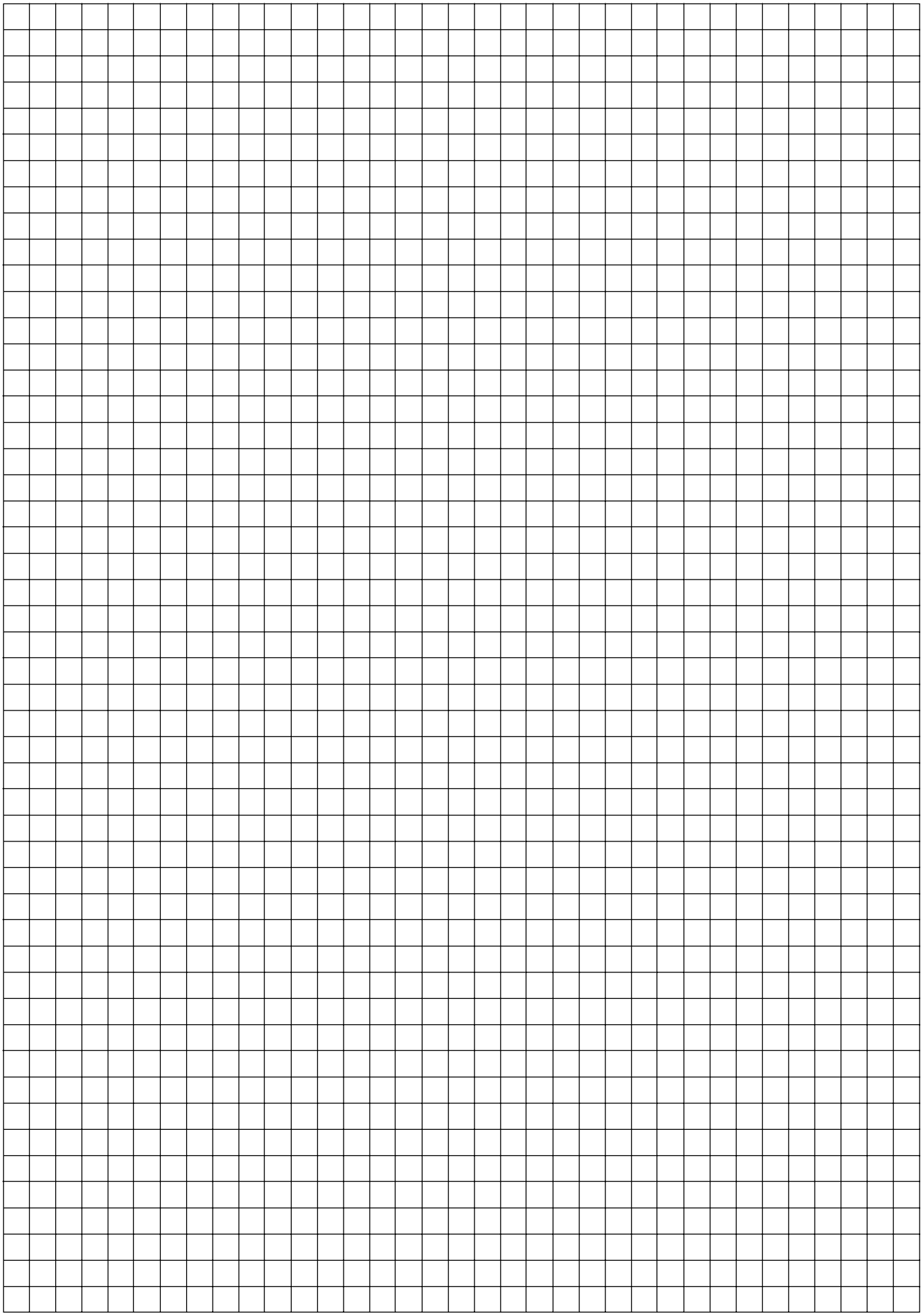

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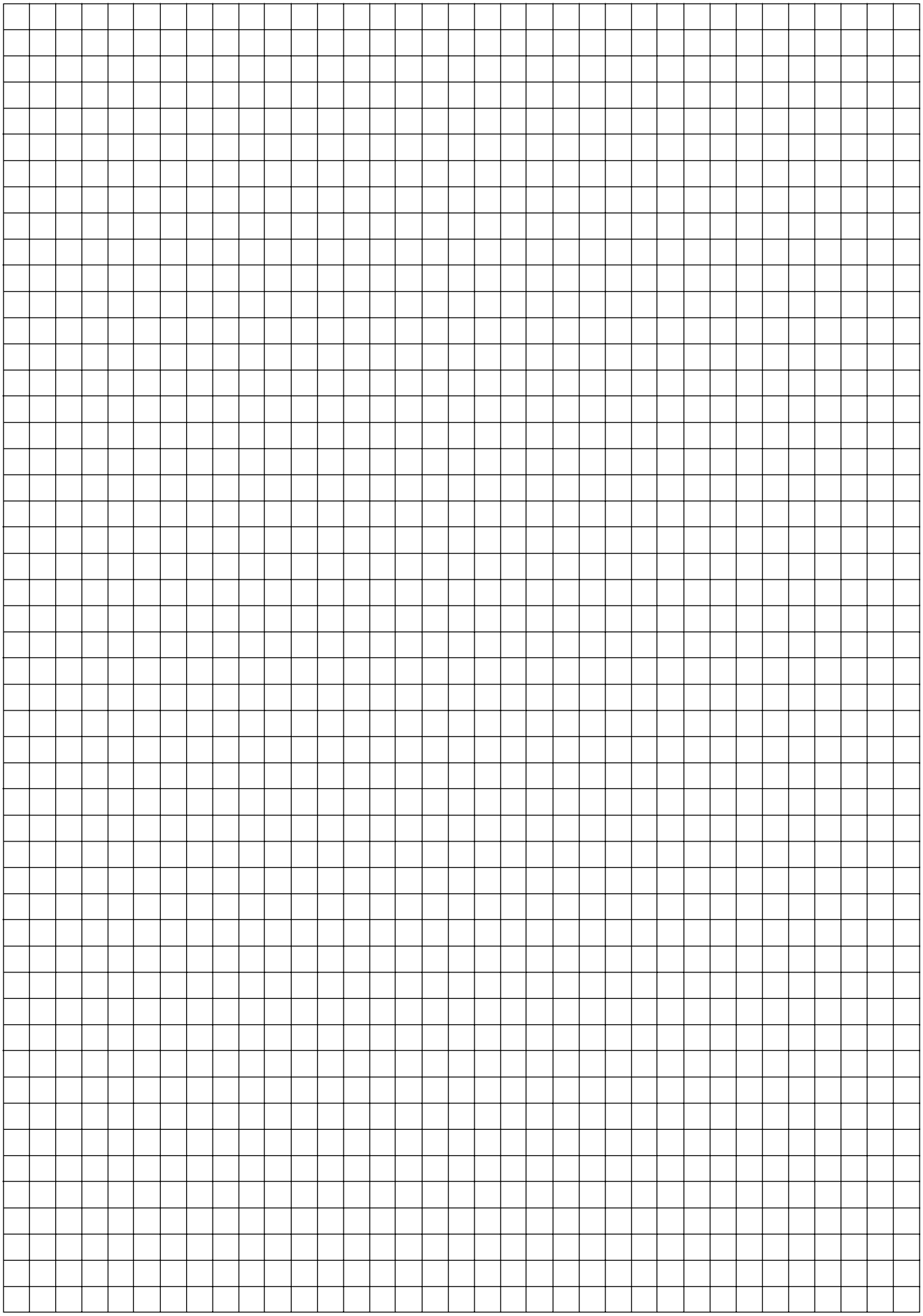

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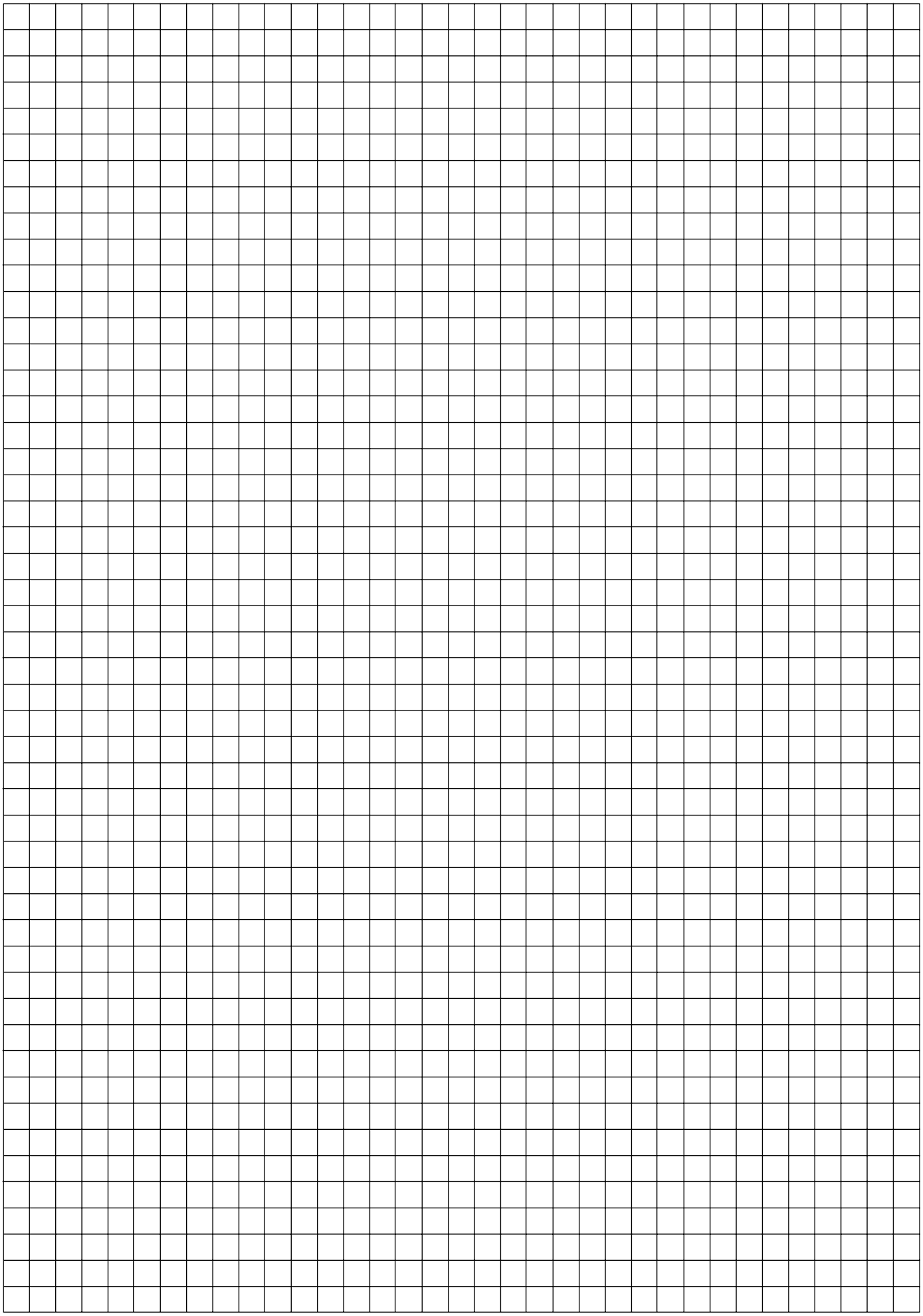

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June 2016

Algebra II Regents

And

Answers

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name:_____

School Name:_____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

(1) $\dfrac{1}{\left(\sqrt[d]{3b}\right)^2}$

(2) $\left(\sqrt{3b}\right)^d$

(3) $\dfrac{1}{\sqrt{3b^d}}$

(4) $\left(\sqrt[d]{3b}\right)^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, T, are left in the semester?

(1) $\dfrac{255 + 93T}{3T} = 90$

(2) $\dfrac{255 + 90T}{3T} = 93$

(3) $\dfrac{255 + 93T}{T + 3} = 90$

(4) $\dfrac{255 + 90T}{T + 3} = 93$

3 Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

(1) $y^2 - 4yi + 4$

(2) $-y^2 - 4yi + 4$

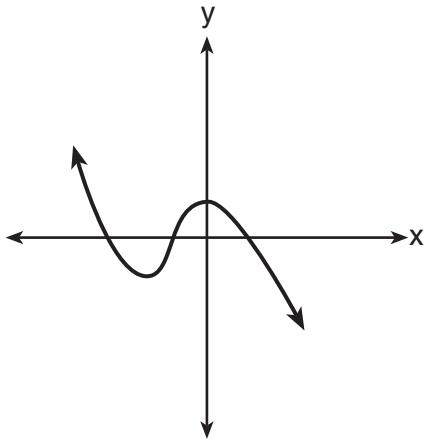
(3) $-y^2 + 4$

(4) $y^2 + 4$

Use this space for computations.

4 Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



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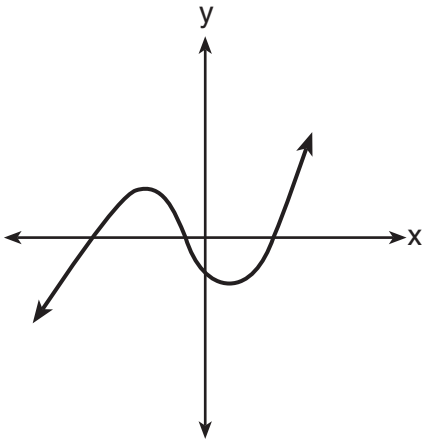

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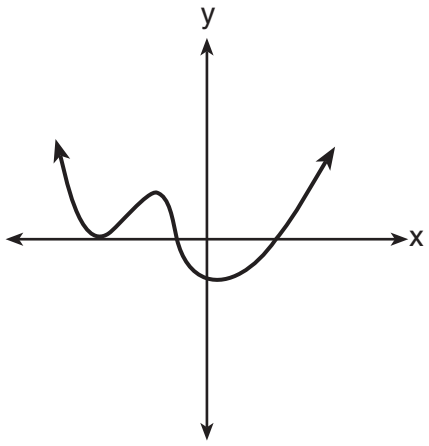

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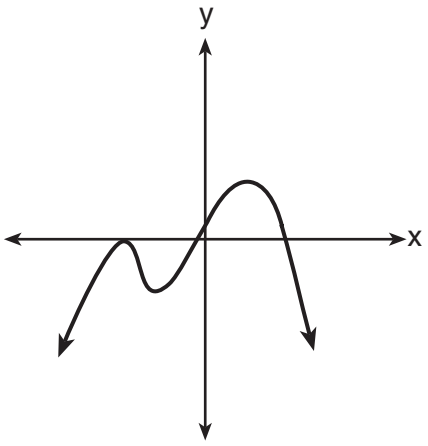


(4)

5 The solution set for the equation $\sqrt{56 - x} = x$ is

(1) $\{-8,7\}$

(2) $\{-7,8\}$

(3) $\{7\}$

(4) $\{\ \}$

Use this space for computations.

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

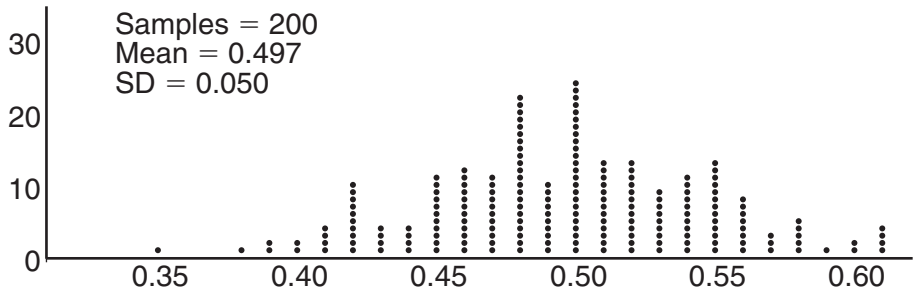
(1) $\{0,\pm 3,4\}$

(2) $\{0,3,4\}$

(3) $\{0,\pm 3,-4\}$

(4) $\{0,3,-4\}$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

(1) 73 of the computer's next 100 coin flips will be heads.

(2) 50 of her next 100 coin flips will be heads.

(3) Her coin is not fair.

(4) Her coin is fair.

8 If $g(c) = 1 - c^2$ and $m(c) = c + 1$, then which statement is *not* true?

(1) $g(c) \bullet m(c) = 1 + c - c^2 - c^3$

(2) $g(c) + m(c) = 2 + c - c^2$

(3) $m(c) - g(c) = c + c^2$

(4) $\frac{m(c)}{g(c)} = \frac{-1}{1-c}$

Use this space for computations.

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent,* is

(1) 6
(2) 48
(3) 68
(4) 95

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of $3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

Use this space for computations.

12 A solution of the equation $2x^2 + 3x + 2 = 0$ is

(1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

(2) $-\frac{3}{4} + \frac{7}{4}i$

(3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

(4) $\frac{1}{2}$

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150

(2) 70

(3) 10

(4) 0

14 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

(1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$

(2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$

(3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$

(4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

15 Which function represents exponential decay?

(1) $y = 2^{0.3t}$

(2) $y = 1.2^{3t}$

(3) $y = \left(\frac{1}{2}\right)^{-t}$

(4) $y = 5^{-t}$

Use this space for computations.

16 Given $f^{-1}(x) = -\frac{3}{4}x + 2$, which equation represents $f(x)$?

(1) $f(x) = \frac{4}{3}x - \frac{8}{3}$

(2) $f(x) = -\frac{4}{3}x + \frac{8}{3}$

(3) $f(x) = \frac{3}{4}x - 2$

(4) $f(x) = -\frac{3}{4}x + 2$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

(1) $-\frac{3}{5}$

(2) $-\frac{3}{4}$

(3) $\frac{3}{5}$

(4) $\frac{4}{5}$

18 Which statement about the graph of $c(x) = \log_6 x$ is *false*?

(1) The asymptote has equation $y = 0$.

(2) The graph has no y-intercept.

(3) The domain is the set of positive reals.

(4) The range is the set of all real numbers.

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

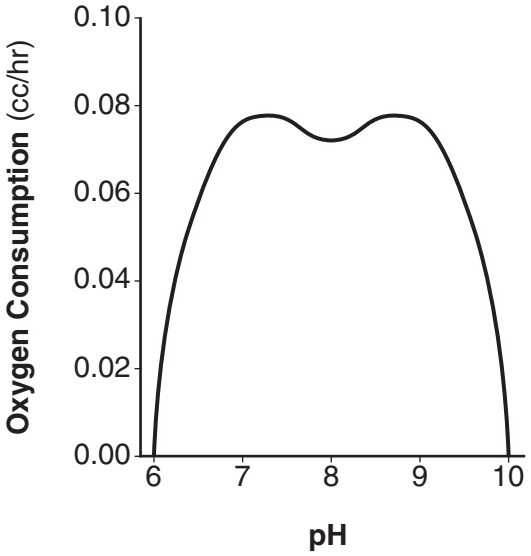
(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

Use this space for computations.

20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

(1) The degree of the polynomial is even.

(2) There is a positive leading coefficient.

(3) At two pH values, there is a relative maximum value.

(4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

Use this space for computations.

22 Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

(1) -3.9

(2) -1.1

(3) 2.1

(4) 4.7

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$

(2) $V = 120 \sin(60t)$

(3) $V = 120 \sin(60\pi t)$

(4) $V = 120 \sin(120\pi t)$

Part II

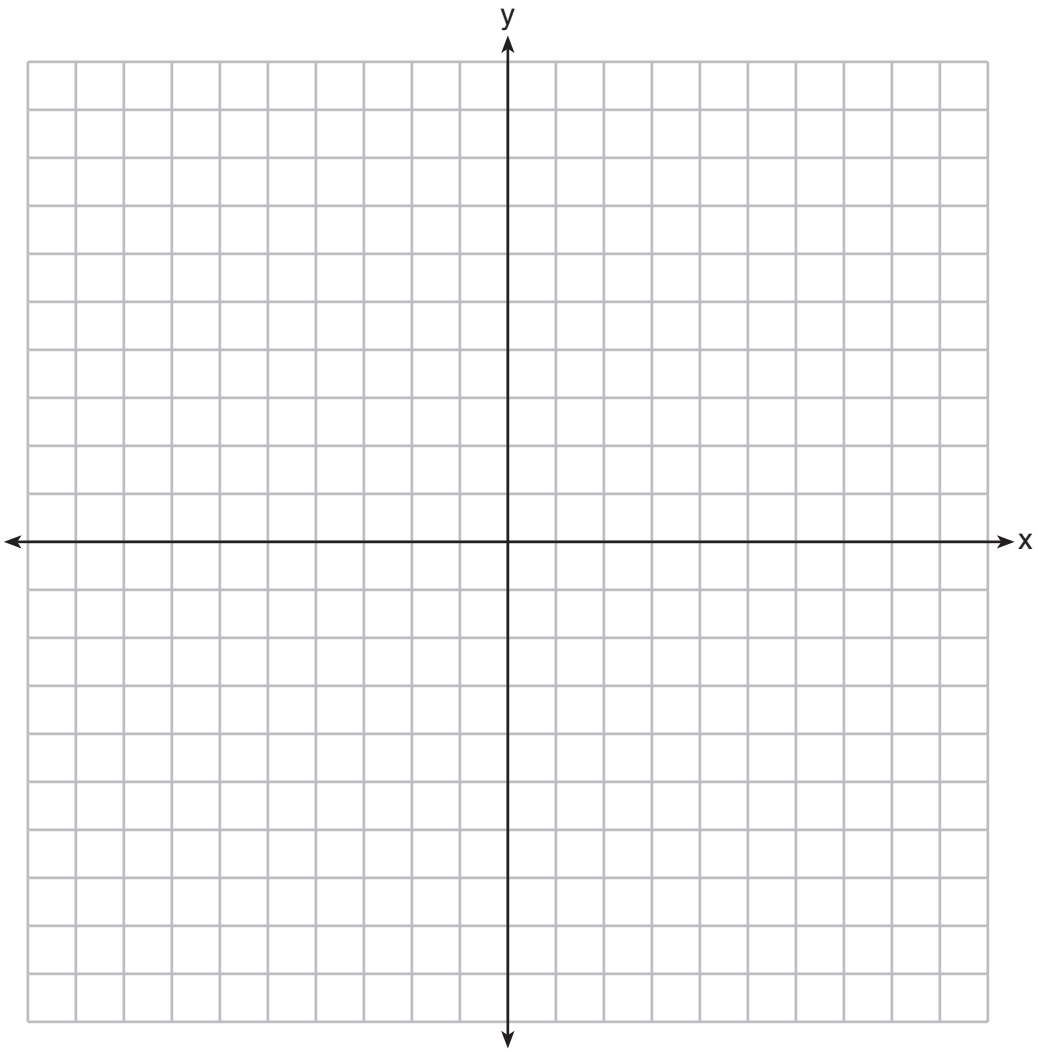
Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

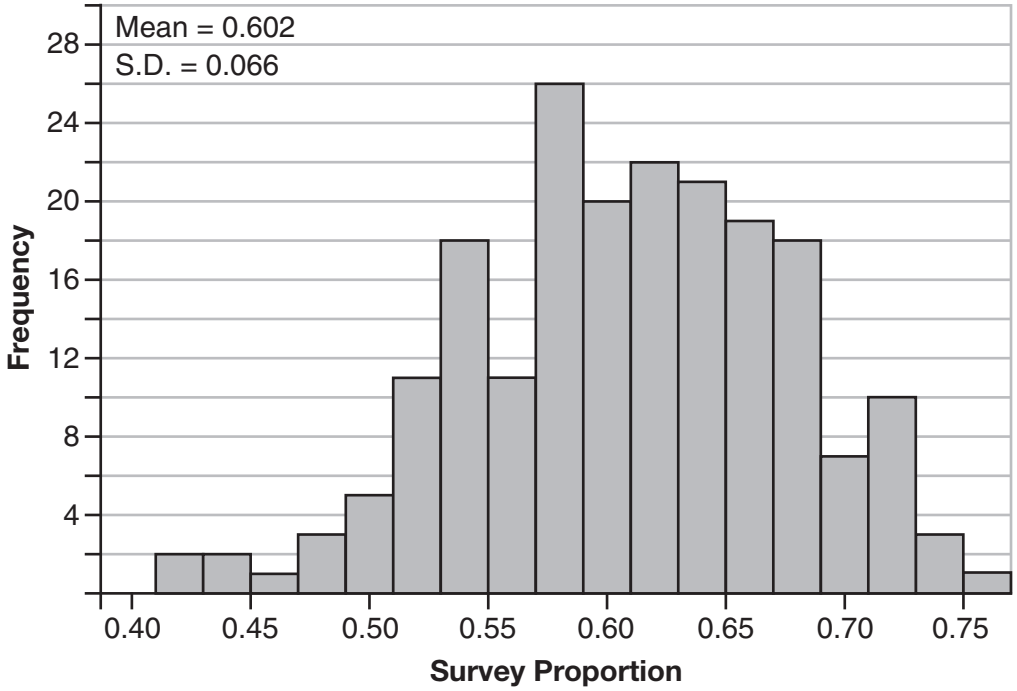
34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

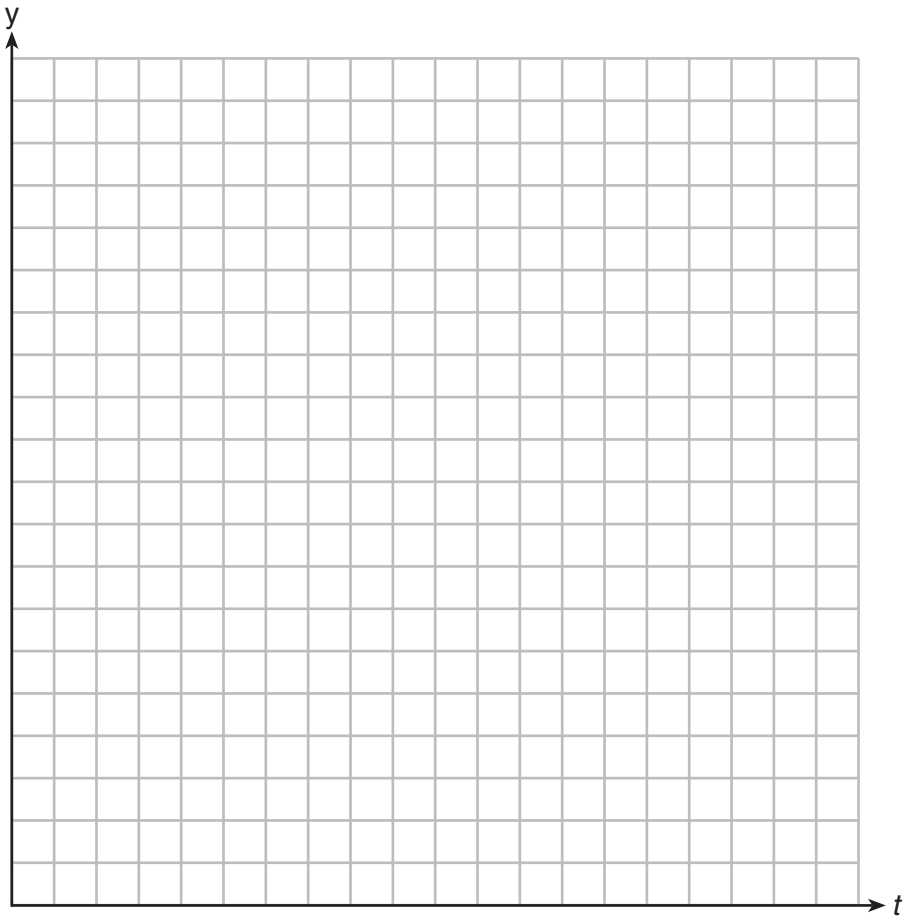
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

Graph each function on the set of axes below.



To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

Part I

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(3) $\frac{1}{\sqrt{3b^d}}$

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2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, T, are left in the semester? 85 x 3

(1) $\frac{255 + 93T}{3T} = 90$

(2) $\frac{255 + 90T}{3T} = 93$

(3) $\frac{255 + 93T}{T+3} = 90$

(4) $\frac{255 + 90T}{T+3} = 93$

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(1) $y^2 - 4yi + 4$

(2) $-y^2 - 4yi + 4$

(3) $-y^2 + 4$

(4) $y^2 + 4$

$(2-yi)(2-yi)$

$4 - 4yi + y^2 i^2$

$-y^2 - 4yi + 4$

Use this space for computations.

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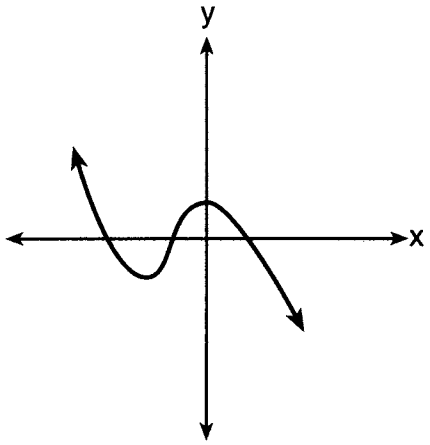
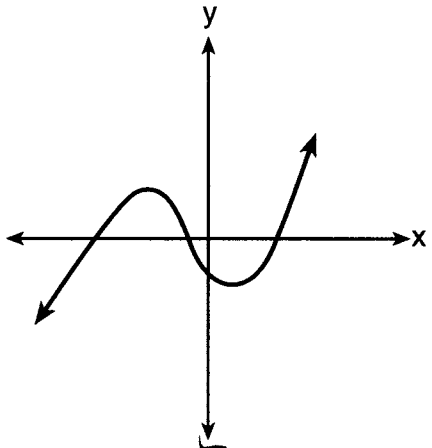
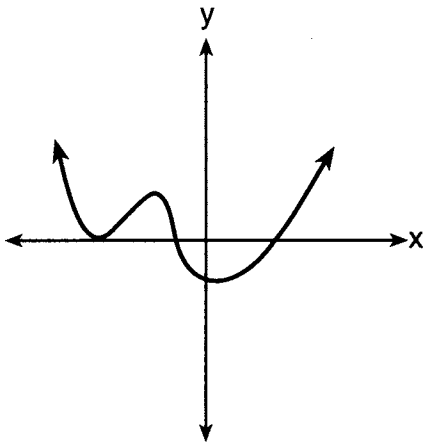


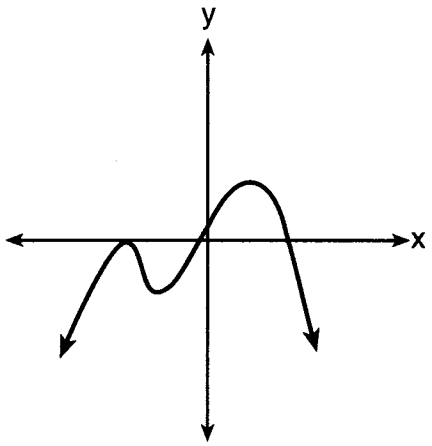
(1)



(3)



(2)



(4)

5 The solution set for the equation $\sqrt{56 - x} = x$ is

(1) $\{-8,7\}$
(2) $\{-7,8\}$
(3) $\{7\}$
(4) $\{\ \}$

$56 - x = x^2$
$0 = x^2 + x - 56$
$0 = (x+8)(x-7)$
$x = -8, 7$
extraneous

Use this space for computations.

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

(1) $\{0,\pm 3,4\}$

(2) $\{0,3,4\}$

(3) $\{0,\pm 3,-4\}$

(4) $\{0,3,-4\}$

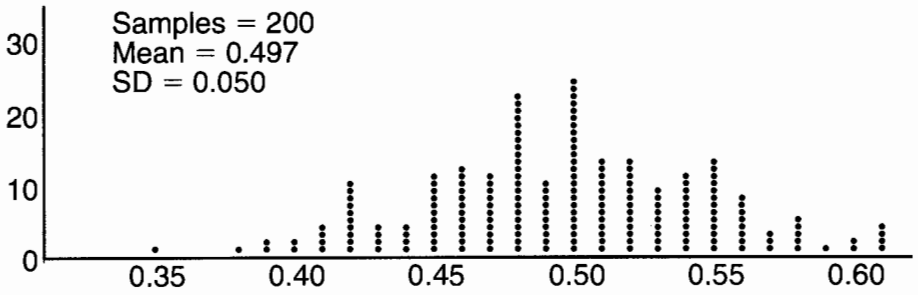
$x(x^3-4x^2-9x+36)=0$

$x(x^2(x-4)-9(x-4))=0$

$x(x^2-9)(x-4)=0$

$x(x+3)(x-3)(x-4)=0$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

(1) 73 of the computer's next 100 coin flips will be heads.

(2) 50 of her next 100 coin flips will be heads.

(3) Her coin is not fair.

(4) Her coin is fair.

8 If $g(c) = 1 - c^2$ and $m(c) = c + 1$, then which statement is *not* true?

(1) $g(c) \cdot m(c) = 1 + c - c^2 - c^3$

(2) $g(c) + m(c) = 2 + c - c^2$

(3) $m(c) - g(c) = c + c^2$

(4) $\frac{m(c)}{g(c)} = \frac{-1}{1-c}$

$\frac{m(c)}{g(c)} = \frac{c+1}{1-c^2} = \frac{c+1}{(1-c)(1+c)} = \frac{1}{1-c}$

Use this space for computations.

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

(1) 6
(2) 48
(3) 68
(4) 95

$z = \frac{69.5 - 64}{2.75} = 2$

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

$P(R) = P(R \mid S)$

Use this space for computations.

12 A solution of the equation $2x^2 + 3x + 2 = 0$ is

(1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

(2) $-\frac{3}{4} + \frac{7}{4}i$

(3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

(4) $\frac{1}{2}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)}$

$\frac{-3 \pm \sqrt{-7}}{4}$

$-\frac{3}{4} \pm \frac{i\sqrt{7}}{4}$

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150

(2) 70

(3) 10

(4) 0

$70(-1) + 80$

$-70 + 80 = 10$

14 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

(1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$

(2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$

(3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$

(4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

$2x+3 \overline{)4x^3 + 0x^2 + 5x + 10}$

$2x^2 - 3x + 7$

$4x^3 + 6x^2$

$-6x^2 + 5x$

$-6x^2 - 9x$

$14x + 10$

$14x + 21$

-11

15 Which function represents exponential decay?

(1) $y = 2^{0.3t}$

(2) $y = 1.2^{3t}$

(3) $y = \left(\frac{1}{2}\right)^{-t}$

(4) $y = 5^{-t}$

$\left(\frac{1}{5}\right)^t$

Use this space for computations.

16 Given $f^{-1}(x) = -\frac{3}{4}x + 2$, which equation represents $f(x)$?

(1) $f(x) = \frac{4}{3}x - \frac{8}{3}$

(3) $f(x) = \frac{3}{4}x - 2$

(2) $f(x) = -\frac{4}{3}x + \frac{8}{3}$

(4) $f(x) = -\frac{3}{4}x + 2$

$x = -\frac{3}{4}y + 2$

$-4x = 3y - 8$

$-4x + 8 = 3y$

$-\frac{4}{3}x + \frac{8}{3} = y$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

(1) $-\frac{3}{5}$

(3) $\frac{3}{5}$

(2) $-\frac{3}{4}$

(4) $\frac{4}{5}$

Since θ terminates in Quadrant II, $\cos\theta$ is negative.

$\sqrt{10^2 - 8^2} = 6$ $\quad \frac{-6}{10} = -\frac{3}{5}$

18 Which statement about the graph of $c(x) = \log_6 x$ is *false*?

(1) The asymptote has equation $y = 0$.

(2) The graph has no y-intercept.

(3) The domain is the set of positive reals.

(4) The range is the set of all real numbers.

The asymptote has equation $x = 0$

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

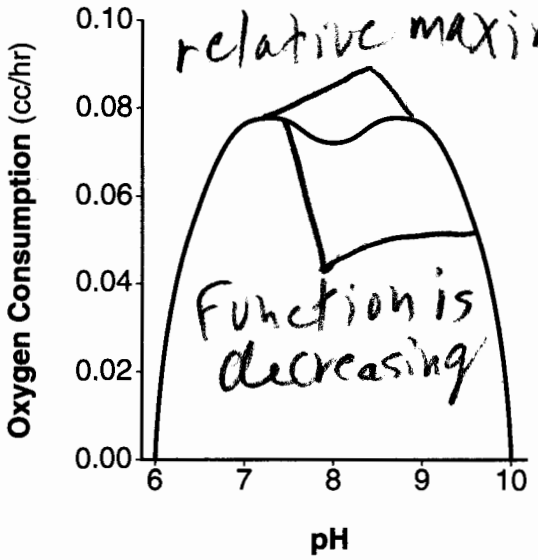
(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 436$

$4(x - 3)^2 + 4(y + 9)^2 = 436$

Use this space for computations.

20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

(1) The degree of the polynomial is even. degree is 4

(2) There is a positive leading coefficient.

(3) At two pH values, there is a relative maximum value.

(4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

$1.0525^{1/12} - 1.00427$

Use this space for computations.

22 Which value, to the *nearest tenth,* is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

(1) -3.9
(2) -1.1
(3) 2.1
(4) 4.7

Use graphing calculator

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$
(2) $V = 120 \sin(60t)$
(3) $V = 120 \sin(60\pi t)$
(4) $V = 120 \sin(120\pi t)$

$\text{period} = \frac{2\pi}{B}$

$\frac{1}{60} = \frac{2\pi}{B}$

$B = 120\pi$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{3-x}{3x} = \frac{-1}{3x}$$

$$3 - x = -1$$

$$4 = x$$

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

Randomly assign participants to two groups, one using toothpaste with ingredient X, one without

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10$

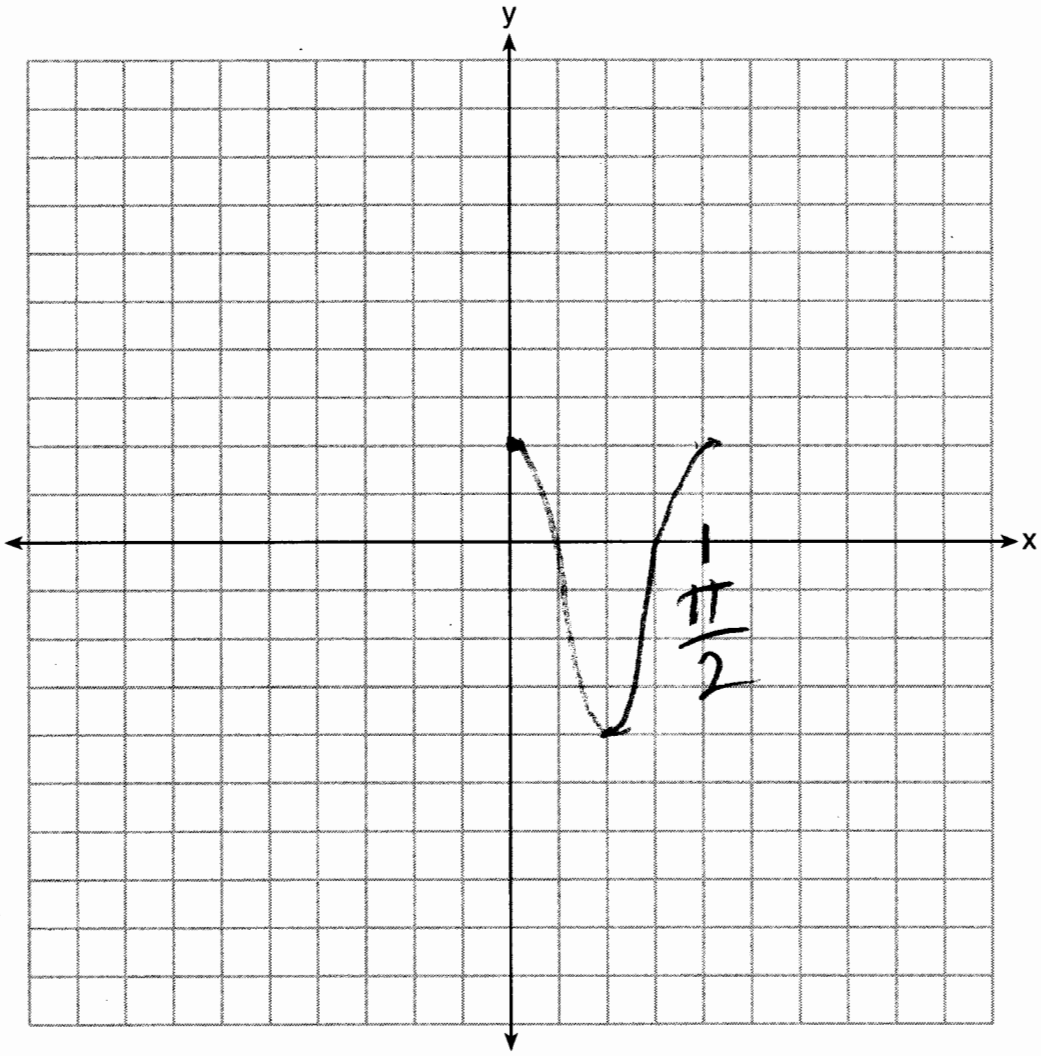
$250 - 100 - 35 - 10$

$105 \neq 0$

Since 5 is not a zero,

$x - 5$ is not a factor

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

649 + 433 − 974 = $\frac{108}{1376}$

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = (x-4)^2$

$\frac{12y}{12} = \frac{(x-4)^2}{12} - \frac{36}{12}$

$y = \frac{1}{12}(x-4)^2 - 3$

The vertex is $(4, -3)$

$(x-4)^2 = 4(3)(y+3)$

$p = 3$

The Focus is $(4, 0)$

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$x^3+8 \overline{)\, x^3+0x^2+0x+9}$$

$$\underline{x^3 \qquad\qquad +8}$$

$$1$$

$$1 + \frac{1}{x^3+8}$$

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$135{,}000 = 100{,}000(1+r)^5$

$\sqrt[5]{1.35} = \sqrt[5]{(1+r)^5}$

$.06 \approx r$

6%

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$

$$2x + 2y = 10$$

$$\frac{2x}{2} + \frac{2y}{2} = \frac{10}{2}$$

$$y = -x + 5$$

$$(x-3)^2 + (-x+5+2)^2 = 16$$

$$x^2 - 6x + 9 + x^2 - 14x + 49 = 16$$

$$2x^2 - 20x + 42 = 0$$

$$x^2 - 10x + 21 = 0$$

$$(x-7)(x-3) = 0$$

$$x = 7 \quad 3$$

$$y = -7 + 5 \qquad y = -3 + 5$$

$$= -2 \qquad y = 2$$

$$(7, -2) \qquad (3, 2)$$

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

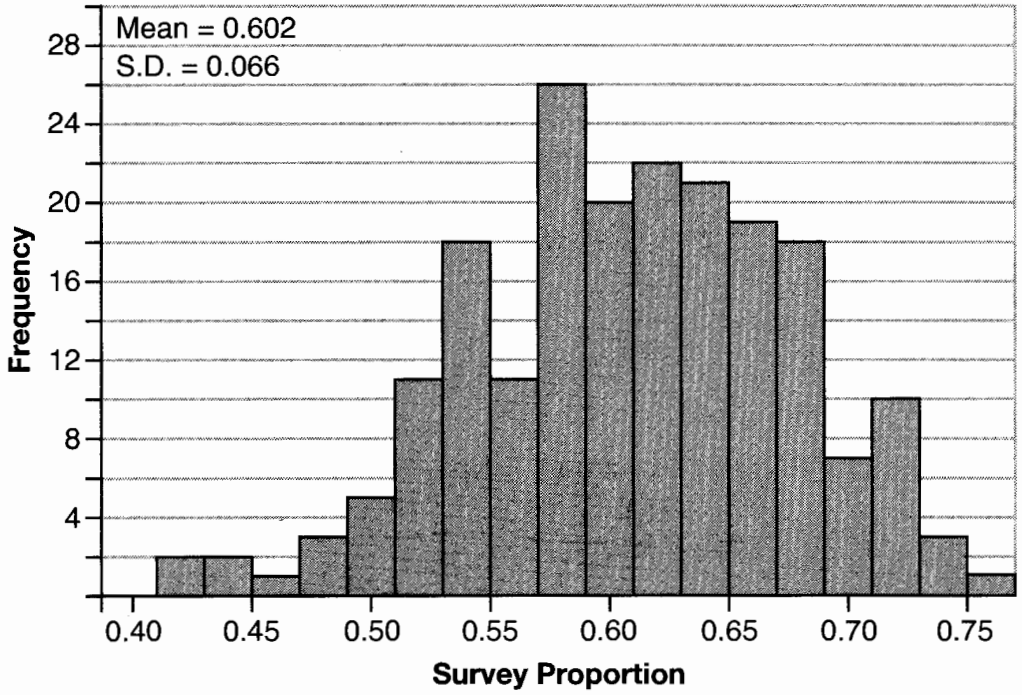
$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660,778.39$$

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

$0.602 \pm 2(0.066)$

$0.47 - 0.73$

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.50 is within this interval, so its possible to get a split vote

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$\frac{g(4) - g(-2)}{4 - -6} = \frac{179 - -49}{6}$

$= 38$

$\frac{f(4) - f(-2)}{4 - -2} = \frac{80 - 1.25}{6} = 13.125$

$g(x)$ has a greater rate of change.

Part IV

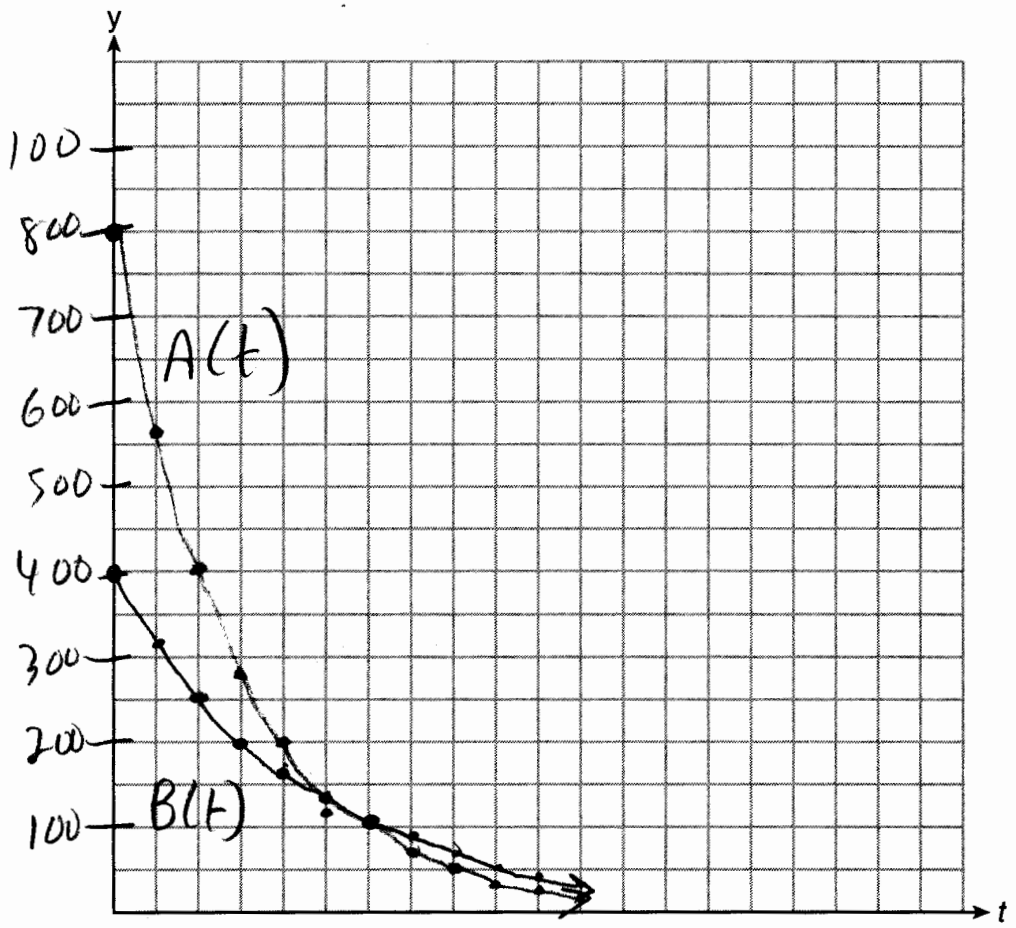
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800e^{-.347t}$ $B(t) = 400e^{-.231t}$

Graph each function on the set of axes below.



To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

6

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$\ln 0.15 = \ln e^{-.347t}$

$\frac{\ln 0.15}{-.347} = \frac{-.347t}{-.347}$

$5.5 \approx t$

June 2017

Algebra II Regents

And

Answers

ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II
(Common Core)

Friday, June 16, 2017 — 1:15 to 4:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II**, **III**, and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

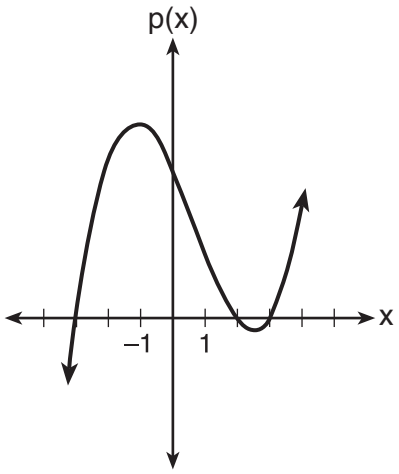
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graph of the function $p(x)$ is sketched below.



Which equation could represent $p(x)$?

(1) $p(x) = (x^2 - 9)(x - 2)$

(2) $p(x) = x^3 - 2x^2 + 9x + 18$

(3) $p(x) = (x^2 + 9)(x - 2)$

(4) $p(x) = x^3 + 2x^2 - 9x - 18$

2 What is the solution to $8(2^{x+3}) = 48$?

(1) $x = \frac{\ln 6}{\ln 2} - 3$

(2) $x = 0$

(3) $x = \frac{\ln 48}{\ln 16} - 3$

(4) $x = \ln 4 - 3$

Use this space for computations.

3 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

(1) interviewing every 5th customer to come into the station

(2) interviewing customers chosen at random by a computer at the checkout

(3) interviewing customers who call an 800 number posted on the customers' receipts

(4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4 The expression $6xi^3(-4xi + 5)$ is equivalent to

(1) $2x - 5i$

(2) $-24x^2 - 30xi$

(3) $-24x^2 + 30x - i$

(4) $26x - 24x^2i - 5i$

5 If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

(1) 1.96

(2) 11.29

(3) $(-0.99, 1.96)$

(4) $(11.29, 32.87)$

6 Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if a and b are positive constants?

(1) right a units, up b units

(2) right a units, down b units

(3) left a units, up b units

(4) left a units, down b units

Use this space for computations.

7 The solution to the equation $4x^2 + 98 = 0$ is

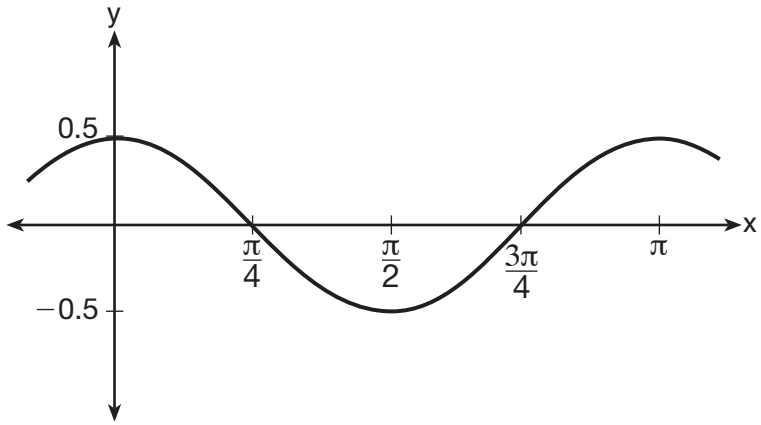
(1) ± 7

(2) $\pm 7i$

(3) $\pm \frac{7\sqrt{2}}{2}$

(4) $\pm \frac{7i\sqrt{2}}{2}$

8 Which equation is represented by the graph shown below?



(1) $y = \frac{1}{2}\cos 2x$

(2) $y = \cos x$

(3) $y = \frac{1}{2}\cos x$

(4) $y = 2\cos \frac{1}{2}x$

9 A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \bullet S(x)$.

The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

(1) $0.15x^3 + 0.02x^2 - 28x + 120$

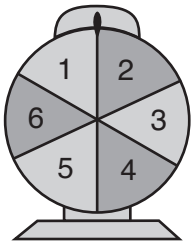
(2) $-0.15x^3 - 0.02x^2 + 28x - 120$

(3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$

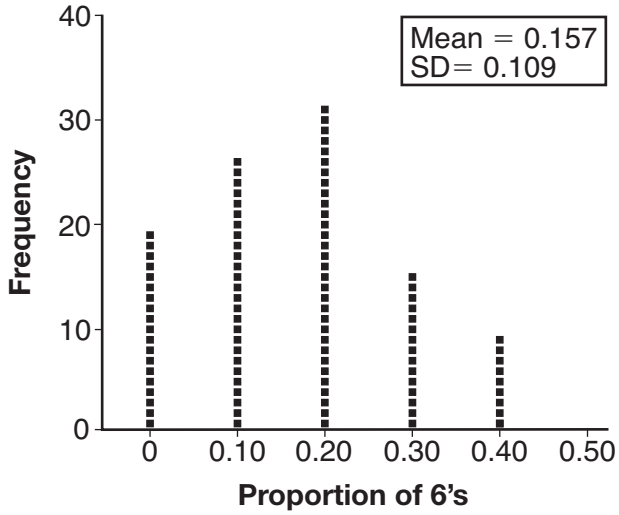
(4) $-0.15x^3 + 32x + 120$

Use this space for computations.

10 A game spinner is divided into 6 equally sized regions, as shown in the diagram below.



For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is $\frac{1}{6}$. The output of the simulation is shown in the diagram below.



Which explanation is appropriate for Miles and his dad to make?

(1) The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.

(2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.

(3) The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.

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Use this space for computations.

11 Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

(1) $x - 2$

(2) $x + 2$

(3) $x - 4$

(4) $x + 4$

12 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$?

(1) $\frac{5 + \sqrt{2}}{5}$

(2) $\frac{\sqrt{23}}{5}$

(3) $\frac{3\sqrt{3}}{5}$

(4) $\frac{\sqrt{35}}{5}$

13 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

(1) $P = 714(0.6500)^y$

(2) $P = 714(0.8500)^y$

(3) $P = 714(0.9716)^y$

(4) $P = 714(0.9750)^y$

14 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive

(1) I, only

(2) II, only

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Use this space for computations.

15 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$.

Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

(1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.

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16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of x?

I. $\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$ II. $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ III. $x^{-\frac{1}{6}}$

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

17 A parabola has its focus at (1,2) and its directrix is $y = -2$. The equation of this parabola could be

(1) $y = 8(x + 1)^2$

(2) $y = \frac{1}{8}(x + 1)^2$

(3) $y = 8(x - 1)^2$

(4) $y = \frac{1}{8}(x - 1)^2$

Use this space for computations.

18 The function $p(t) = 110e^{0.03922t}$ models the population of a city, in millions, t years after 2010. As of today, consider the following two statements:

I. The current population is 110 million.
II. The population increases continuously by approximately 3.9% per year.

This model supports

(1) I, only
(2) II, only
(3) both I and II
(4) neither I nor II

19 To solve $\frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2 - 2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

(1) 2 is an extraneous solution.
(2) $\frac{7}{2}$ is an extraneous solution.
(3) 0 and 2 are extraneous solutions.
(4) This equation does not contain any extraneous solutions.

20 Given $f(9) = -2$, which function can be used to generate the sequence $-8,-7.25,-6.5,-5.75,...$?

(1) $f(n) = -8 + 0.75n$
(2) $f(n) = -8 - 0.75(n - 1)$
(3) $f(n) = -8.75 + 0.75n$
(4) $f(n) = -0.75 + 8(n - 1)$

21 The function $f(x) = 2^{-0.25x} \cdot \sin\left(\frac{\pi}{2}x\right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7,7]$, to the *nearest hundredth*?

(1) -3.66
(2) -0.30
(3) -0.26
(4) 3.36

Use this space for computations.

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is \$329.99. If she plans to run the unit three months out of the year for an annual operating cost of \$108.78, which function models the cost per year over the lifetime of the unit, $C(n)$, in terms of the number of years, n, that she owns the air conditioner?

(1) $C(n) = 329.99 + 108.78n$

(2) $C(n) = 329.99 + 326.34n$

(3) $C(n) = \dfrac{329.99 + 108.78n}{n}$

(4) $C(n) = \dfrac{329.99 + 326.34n}{n}$

23 The expression $\dfrac{-3x^2 - 5x + 2}{x^3 + 2x^2}$ can be rewritten as

(1) $\dfrac{-3x - 3}{x^2 + 2x}$

(2) $\dfrac{-3x - 1}{x^2}$

(3) $-3x^{-1} + 1$

(4) $-3x^{-1} + x^{-2}$

24 Jasmine decides to put \$100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, S, will Jasmine have after one year?

(1) $S = 100(1.03)^{12}$

(2) $S = \dfrac{100 - 100\left(1.0025\right)^{12}}{1 - 1.0025}$

(3) $S = 100(1.0025)^{12}$

(4) $S = \dfrac{100 - 100\left(1.03\right)^{12}}{1 - 1.03}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

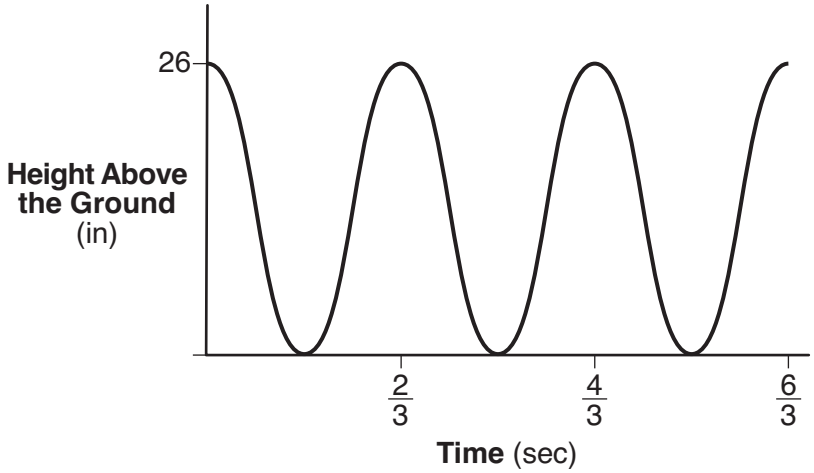
25 Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$.

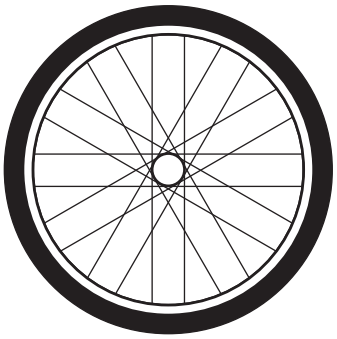
What does your answer tell you about $x - 2$ as a factor of $r(x)$? Explain.

26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.

27 Over the set of integers, factor the expression $4x^3 - x^2 + 16x - 4$ completely.

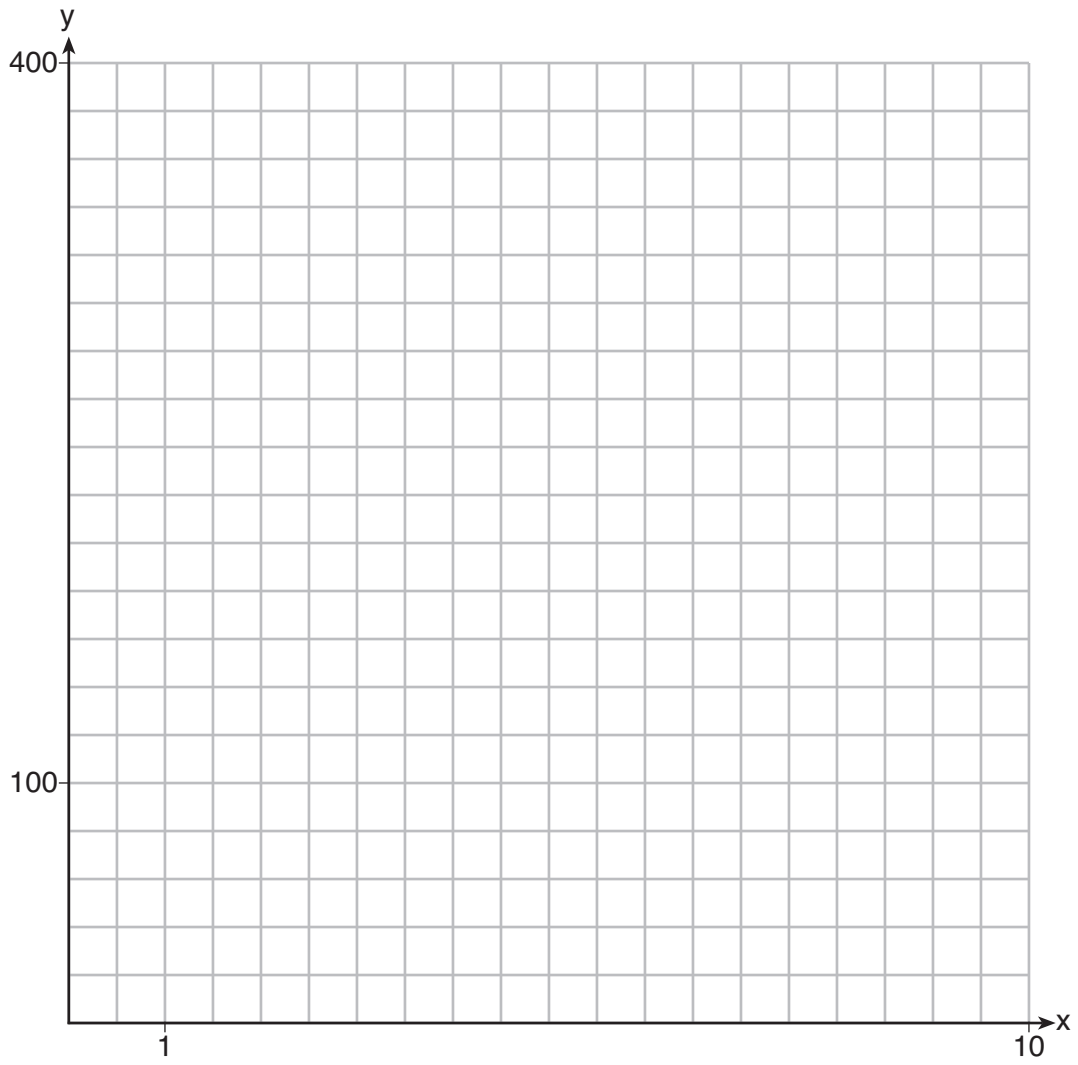
28 The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.





Identify the period of the graph and describe what the period represents in this context.

29 Graph $y = 400(.85)^{2x} - 6$ on the set of axes below.



30 Solve algebraically for all values of x:

$$\sqrt{x-4}+x=6$$

31 Write $\sqrt[3]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.

32 Data collected about jogging from students with two older siblings are shown in the table below.

	Neither Sibling Jogs	One Sibling Jogs	Both Siblings Jog
Student Does Not Jog	1168	1823	1380
Student Jogs	188	416	400

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the following system of equations algebraically for all values of x, y, and z:

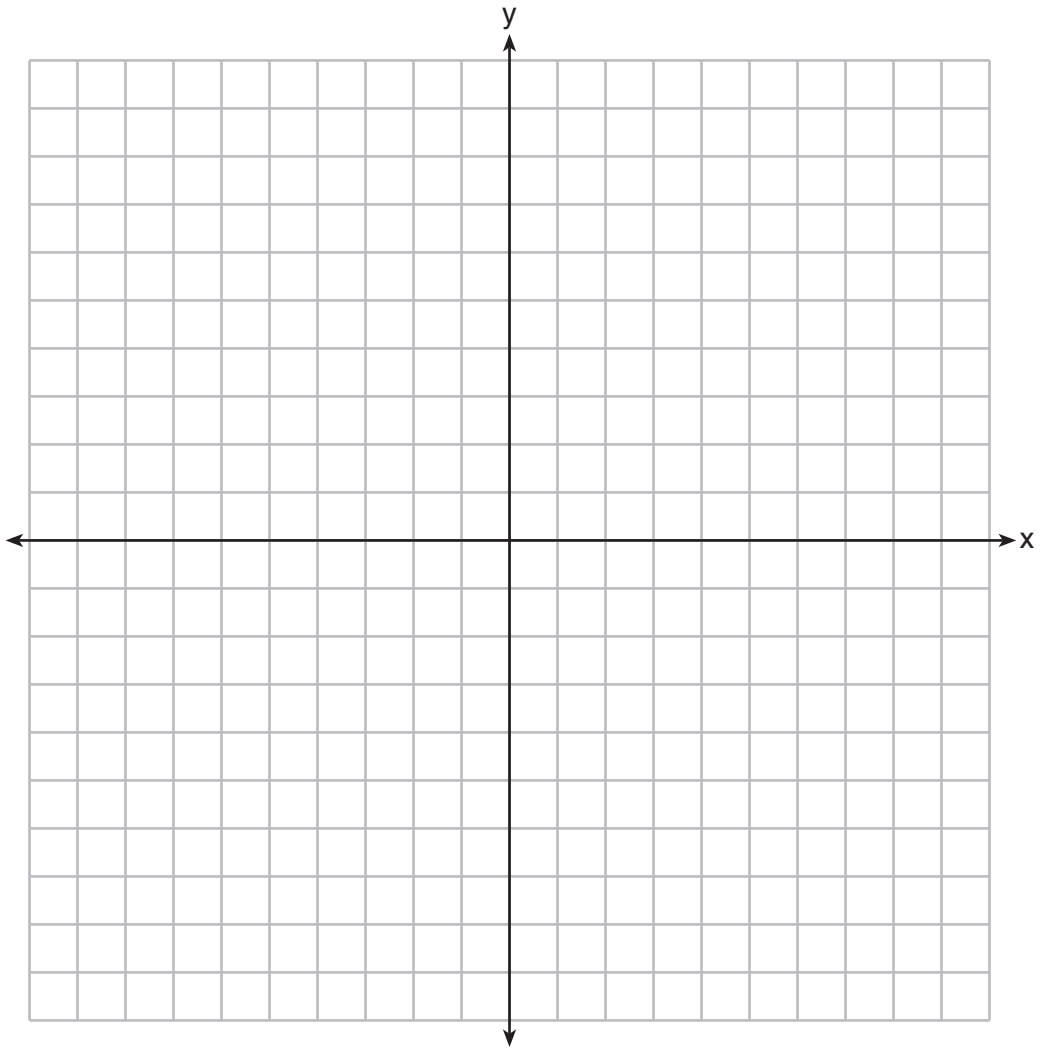
$$x + y + z = 1$$
$$2x + 4y + 6z = 2$$
$$-x + 3y - 5z = 11$$

34 Jim is looking to buy a vacation home for \$172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage.

With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

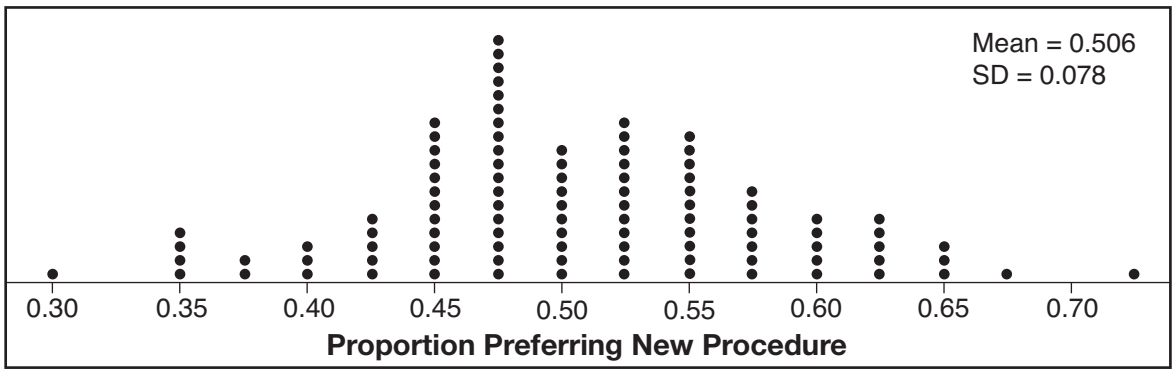
Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$1100.

35 Graph $y = \log_2(x + 3) - 5$ on the set of axes below. Use an appropriate scale to include *both* intercepts.



Describe the behavior of the given function as x approaches -3 and as x approaches positive infinity.

36 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.



Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the *nearest hundredth*.

Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides *not* to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m.

Using this equation, solve for h, to the *nearest ten thousandth*.

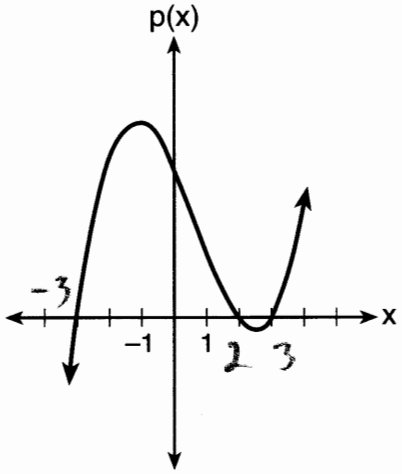
Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graph of the function $p(x)$ is sketched below.



Which equation could represent $p(x)$?

(1) $p(x) = (x^2 - 9)(x - 2)$

(2) $p(x) = x^3 - 2x^2 + 9x + 18$

(3) $p(x) = (x^2 + 9)(x - 2)$

(4) $p(x) = x^3 + 2x^2 - 9x - 18$

$(x+3)(x-3)(x-2)$

$x = -3 \quad 3 \quad 2$

2 What is the solution to $8(2^{x+3}) = 48$?

(1) $x = \frac{\ln 6}{\ln 2} - 3$

(2) $x = 0$

(3) $x = \frac{\ln 48}{\ln 16} - 3$

(4) $x = \ln 4 - 3$

$\ln 2^{x+3} = \ln 6$

$(x+3)\ln 2 = \ln 6$

$x = \frac{\ln 6}{\ln 2} - 3$

Use this space for computations.

3 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

(1) interviewing every 5th customer to come into the station

(2) interviewing customers chosen at random by a computer at the checkout

(3) interviewing customers who call an 800 number posted on the customers' receipts self selection

(4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4 The expression $6xi^3(-4xi + 5)$ is equivalent to

(1) $2x - 5i$

(2) $-24x^2 - 30xi$

(3) $-24x^2 + 30x - i$

(4) $26x - 24x^2i - 5i$

$-24x^2i^4 + 30xi^3$

$-24x^2(1) + 30x(-1)$

5 If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

(1) 1.96

(2) 11.29

(3) $(-0.99, 1.96)$

(4) $(11.29, 32.87)$

6 Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if a and b are positive constants?

(1) right a units, up b units

(2) right a units, down b units

(3) left a units, up b units

(4) left a units, down b units

Use this space for computations.

7 The solution to the equation $4x^2 + 98 = 0$ is

(1) ± 7

(2) $\pm 7i$

(3) $\pm \frac{7\sqrt{2}}{2}$

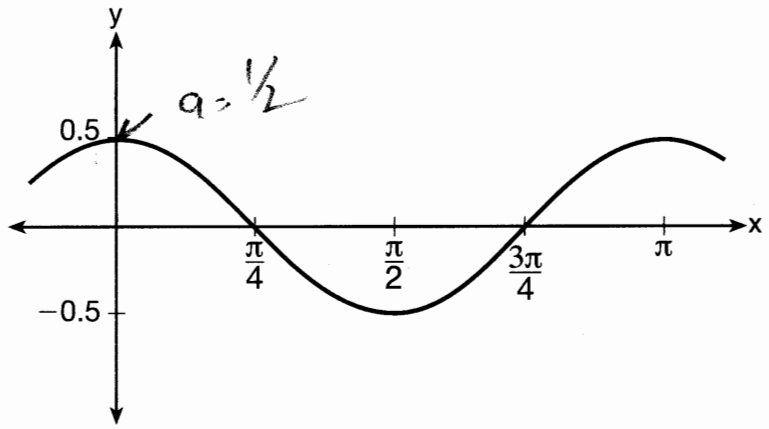
(4) $\pm \frac{7i\sqrt{2}}{2}$

$4x^2 = -98$

$x^2 = \frac{-49}{2}$

$x = \pm\sqrt{\frac{-49}{2}}\ \frac{\sqrt{2}}{\sqrt{2}}$

8 Which equation is represented by the graph shown below?



$a = 1/2$

$\text{period} = \frac{2\pi}{b}$

$\pi = \frac{2\pi}{b}$

$b = 2$

(1) $y = \frac{1}{2}\cos 2x$

(2) $y = \cos x$

(3) $y = \frac{1}{2}\cos x$

(4) $y = 2\cos \frac{1}{2}x$

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The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

(1) $0.15x^3 + 0.02x^2 - 28x + 120$

(2) $-0.15x^3 - 0.02x^2 + 28x - 120$

(3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$

(4) $-0.15x^3 + 32x + 120$

$x \cdot S(x) - C(x)$

$x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120)$

$30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120$

Use this space for computations.

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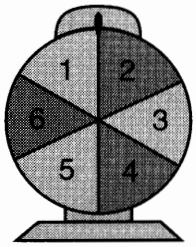

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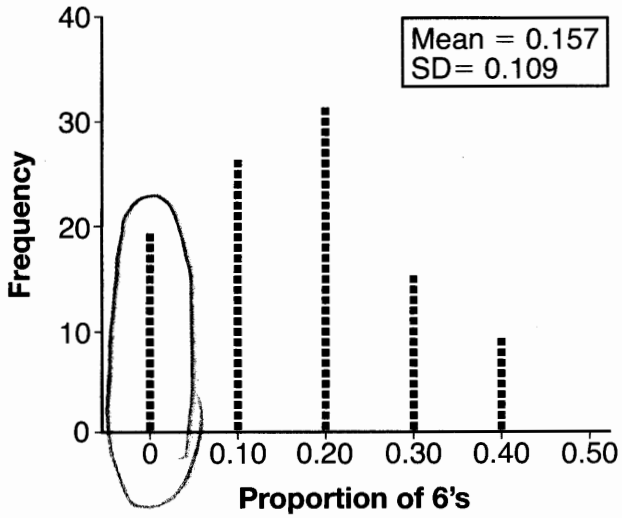

$\frac{1}{6} = .166$

0 falls within 2 standard deviations of the mean

Which explanation is appropriate for Miles and his dad to make?

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(2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.

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(3) $x - 4$
(4) $x + 4$

$$\begin{array}{c|ccccc} 2 & 1 & 0 & -4 & -4 & 8 \\ & & 2 & 4 & 0 & -8 \\ \hline & 1 & 2 & 0 & -4 & 0 \end{array}$$

12 Given that $\sin^2\theta + \cos^2\theta = 1$ and $\sin\theta = -\frac{\sqrt{2}}{5}$, what is a possible value of $\cos\theta$?

(1) $\frac{5+\sqrt{2}}{5}$
(2) $\frac{\sqrt{23}}{5}$
(3) $\frac{3\sqrt{3}}{5}$
(4) $\frac{\sqrt{35}}{5}$

$\cos\theta = \pm\sqrt{1-\left(\frac{-\sqrt{2}}{5}\right)^2}$

$= \pm\sqrt{\frac{25}{25} - \frac{2}{25}} = \pm\frac{\sqrt{23}}{5}$

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(2) $P = 714(0.8500)^y$
(3) $P = 714(0.9716)^y$
(4) $P = 714(0.9750)^y$

$(.75)^{1/10} \approx .9716$

14 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

I: dependent
II: independent
III: mutually exclusive

(1) I, only
(2) II, only
(3) I and III
(4) II and III

If independent,

$P(A \text{ and } B) = P(A) \cdot P(B)$

$.125 = .5 \cdot .25$ ✓

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = .25 + .5 - .125$

$= .625$

If mutually exclusive

$P(A \text{ or } B) = P(A) + P(B)$

$.625 = .5 + .25$ No

Use this space for computations.

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Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

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(2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.

(3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.

(4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

RANGE

	BH	PH
MIN	31.386	66.491
MID	55.3	86.729
MAX	79.214	106.967
RANGE	47.828	40.476

16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of x?

$\frac{1}{\sqrt[6]{x}}$

I. $\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$ II. $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ III. $x^{-\frac{1}{6}}$

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

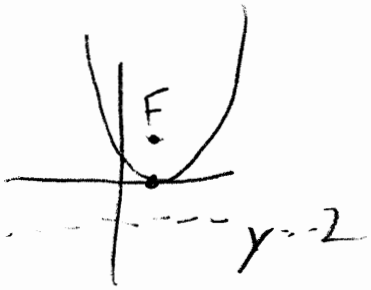
17 A parabola has its focus at (1,2) and its directrix is $y = -2$. The equation of this parabola could be

(1) $y = 8(x + 1)^2$

(2) $y = \frac{1}{8}(x + 1)^2$

(3) $y = 8(x - 1)^2$

(4) $y = \frac{1}{8}(x - 1)^2$



$y = \frac{1}{4p}(x-h)^2 + k$

$y = \frac{1}{8}(x-1)^2 + 0$

Vertex (1,0)

$p = 2$

Use this space for computations.

18 The function $p(t) = 110e^{0.03922t}$ models the population of a city, in millions, t years after 2010. As of today, consider the following two statements:

I. The ~~current~~ population is 110 million.

II. The population increases continuously by approximately 3.9% per year.

This model supports

(1) I, only

(2) II, only

(3) both I and II

(4) neither I nor II

19 To solve $\frac{2x}{x-2} - \frac{11}{x} = \frac{8}{x^2-2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

(1) 2 is an extraneous solution.

(2) $\frac{7}{2}$ is an extraneous solution.

(3) 0 and 2 are extraneous solutions.

(4) This equation does not contain any extraneous solutions.

20 Given $f(9) = -2$, which function can be used to generate the sequence $-8, -7.25, -6.5, -5.75, \ldots$?

(1) $f(n) = -8 + 0.75n$

(2) $f(n) = -8 - 0.75(n - 1)$

(3) $f(n) = -8.75 + 0.75n$

(4) $f(n) = -0.75 + 8(n - 1)$

21 The function $f(x) = 2^{-0.25x} \cdot \sin\left(\frac{\pi}{2}x\right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7,7]$, to the *nearest hundredth*?

(1) -3.66

(2) -0.30

(3) -0.26

(4) 3.36

Use this space for computations.

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is \$329.99. If she plans to run the unit three months out of the year for an annual operating cost of \$108.78, which function models the cost per year over the lifetime of the unit, $C(n)$, in terms of the number of years, n, that she owns the air conditioner.

(1) $C(n) = 329.99 + 108.78n$

(2) $C(n) = 329.99 + 326.34n$

(3) $C(n) = \dfrac{329.99 + 108.78n}{n}$

(4) $C(n) = \dfrac{329.99 + 326.34n}{n}$

23 The expression $\dfrac{-3x^2 - 5x + 2}{x^3 + 2x^2}$ can be rewritten as

(1) $\dfrac{-3x - 3}{x^2 + 2x}$

(2) $\dfrac{-3x - 1}{x^2}$

(3) $-3x^{-1} + 1$

(4) $-3x^{-1} + x^{-2}$

$\dfrac{(-3x+1)(x+2)}{x^2(x+2)}$

$\dfrac{-3x}{x^2} + \dfrac{1}{x^2}$

24 Jasmine decides to put \$100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, S, will Jasmine have after one year?

(1) $S = 100(1.03)^{12}$

(2) $S = \dfrac{100 - 100(1.0025)^{12}}{1 - 1.0025}$

(3) $S = 100(1.0025)^{12}$

(4) $S = \dfrac{100 - 100(1.03)^{12}}{1 - 1.03}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$.

$$\begin{array}{c|cccc} 2 & 1 & -4 & 4 & -6 \\ & & 2 & -4 & 0 \\ \hline & 1 & -2 & 0 & -6 \end{array}$$

$r(2) = -6$

What does your answer tell you about $x - 2$ as a factor of $r(x)$? Explain.

$x-2$ is not a factor since there is a remainder

26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.

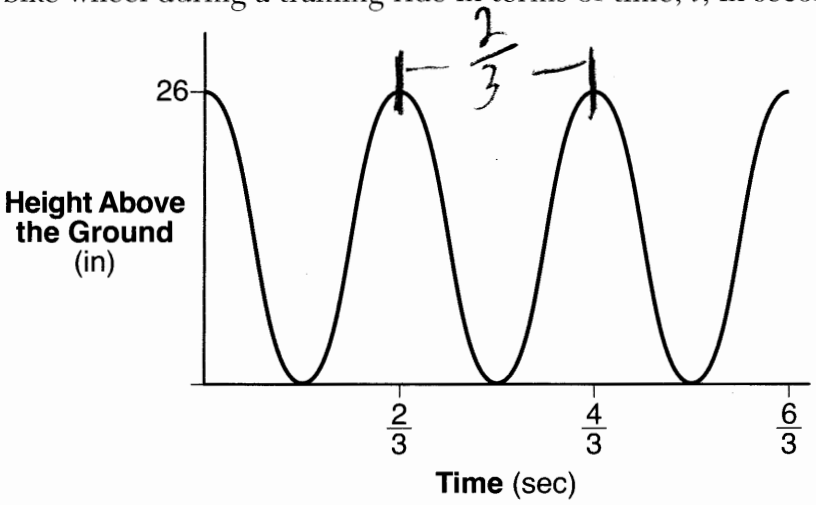
69%, using a graphing calculator

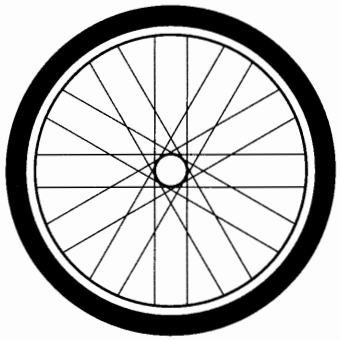
27 Over the set of integers, factor the expression $4x^3 - x^2 + 16x - 4$ completely.

$x^2(4x-1) + 4(4x-1)$

$(x^2+4)(4x-1)$

28 The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.



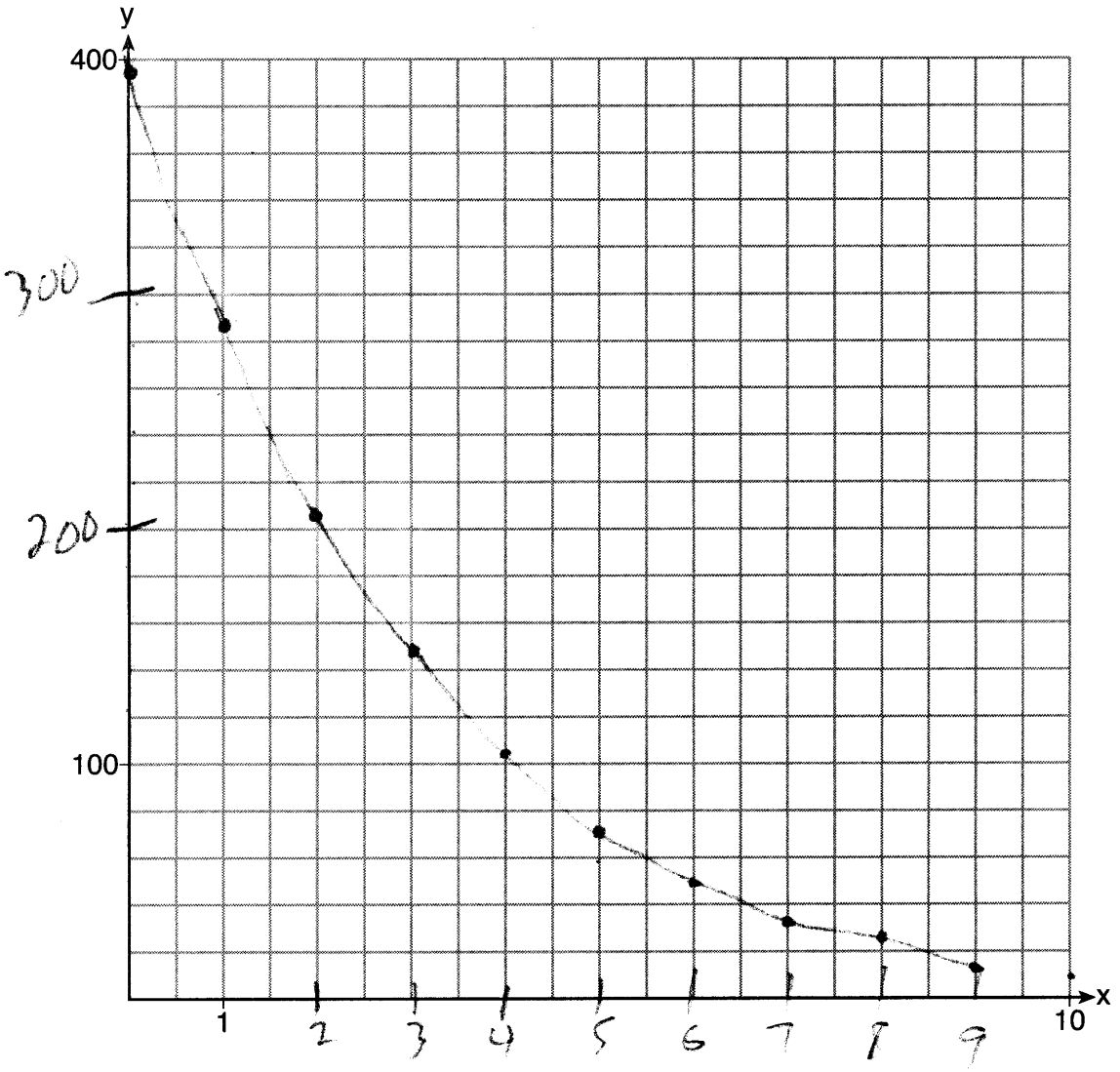


Identify the period of the graph and describe what the period represents in this context.

2/3 sec

The wheel rotates once every 2/3 sec.

29 Graph $y = 400(.85)^{2x} - 6$ on the set of axes below.



30 Solve algebraically for all values of x:

$$\sqrt{x-4}+x=6$$

$\sqrt{x-4} = 6-x$

$x-4 = 36-12x+x^2$

$0 = x^2-13x+40$

$0 = (x-8)(x-5)$

$x = 5, \cancel{8}$

$\sqrt{8-4}+8 \neq 6$

31 Write $\sqrt[3]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.

$x^{1/3} \cdot x^{1/2} = x^{5/6}$

32 Data collected about jogging from students with two older siblings are shown in the table below.

	Neither Sibling Jogs	One Sibling Jogs	Both Siblings Jog
Student Does Not Jog	1168	1823	1380
Student Jogs	188	416	400

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

1 jogs

416

2239

.19

<

both jog

400

1780

.22

more likely

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the following system of equations algebraically for all values of x, y, and z:

$$x + y + z = 1$$
$$2x + 4y + 6z = 2$$
$$-x + 3y - 5z = 11$$

$x+y+z=1$
$-x+3y-5z=11$

$4y-4z=12$

$y-z=3$

$2x+2y+2z=2$
$2x+4y+6z=2$

$2y+4z=0$

$y+2z=0$

$y=-2z$

$-2z-z=3$

$-3z=3$

$z=-1$

$y-(-1)=3$

$y=2$

$x+2-1=1$

$x=0$

$(0,2,-1)$

34 Jim is looking to buy a vacation home for \$172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \cdot \frac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage.

With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

$$M = 172600 \cdot \frac{.00305(1+.00305)^{12\cdot 15}}{(1+.00305)^{12\cdot 15} - 1}$$

$$\approx 1247$$

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be \$1100.

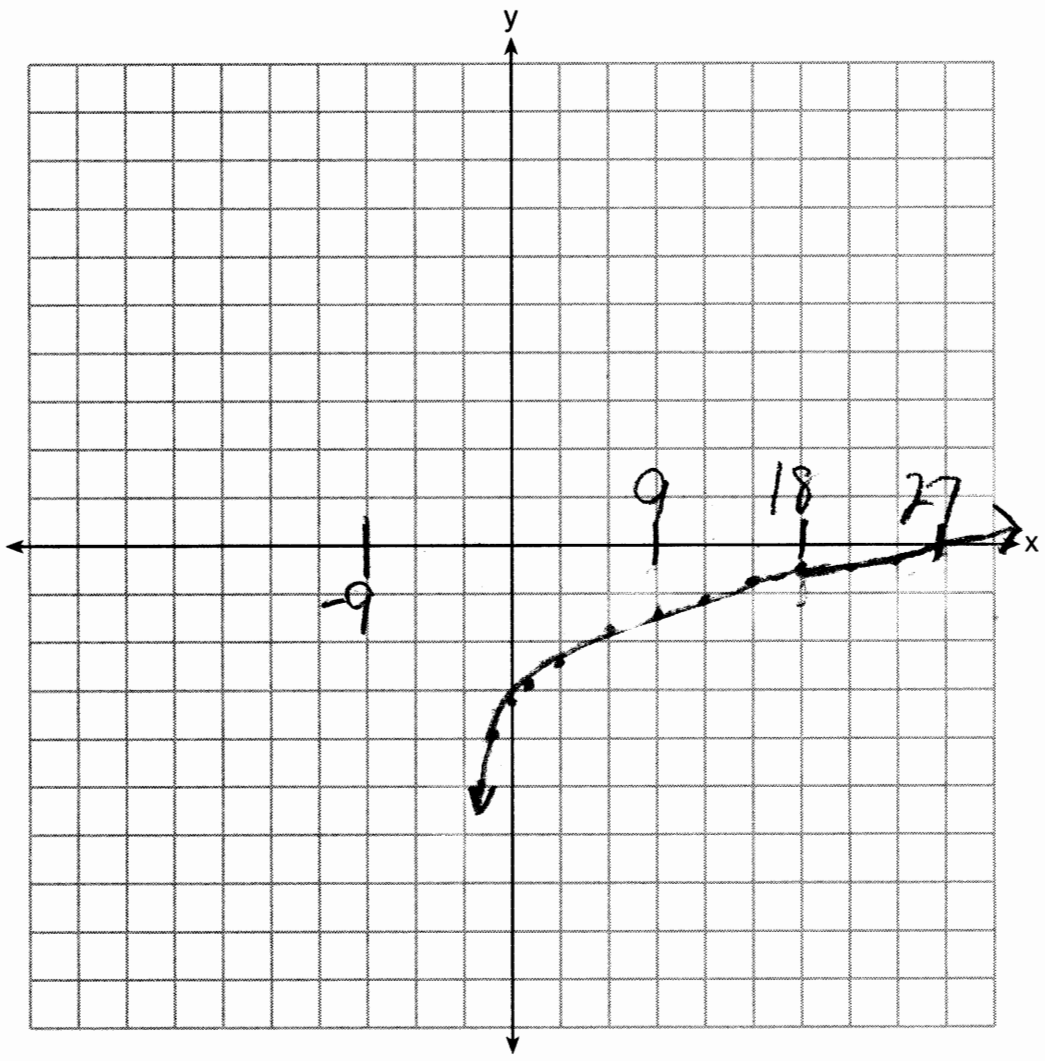
$$1100 = (172{,}600 - x)\left(\frac{.00305(1.00305)^{12\cdot 15}}{(1.00305)^{12\cdot 15} - 1}\right)$$

$$1100 \approx (172600 - x)(.007278)$$

$$152193 \approx 172600 - x$$

$$x \approx 20407$$

35 Graph $y = \log_2(x + 3) - 5$ on the set of axes below. Use an appropriate scale to include *both* intercepts.

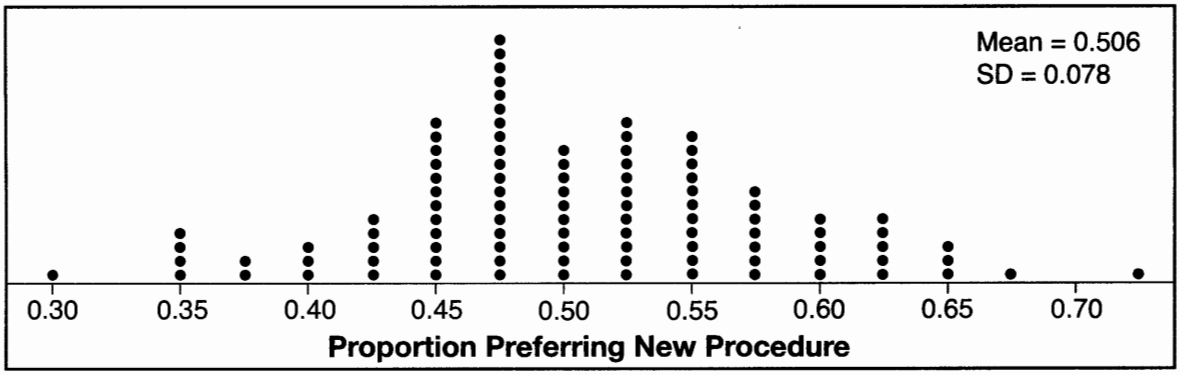


Describe the behavior of the given function as x approaches -3 and as x approaches positive infinity.

As $x \to -3$, $y \to -\infty$

As $x \to \infty$, $y \to \infty$

36 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.



Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the *nearest hundredth*.

$.506 \pm 2(.078)$

$0.35 - 0.66$

Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides *not* to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

The 32.5% value falls below the 95% confidence level

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m.

$100 = 140\left(\frac{1}{2}\right)^{5/h}$

Using this equation, solve for h, to the *nearest ten thousandth*.

$\log \frac{100}{140} = \log \frac{1}{2}^{5/h}$

$\log \frac{5}{7} = \frac{5}{h} \log \frac{1}{2}$

$h = \frac{5 \log 1/2}{\log 5/7} \approx 10.3002$

Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour*.

$40 = 140(.5)^{\frac{t}{10.3002}}$

$\log \frac{2}{7} = \log .5^{t/10.3002}$

$\frac{10.3002 \log 2/7}{\log .5} = t$

≈ 18.6

June 2018

Algebra II Regents

And

Answers

ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, June 14, 2018 — 1:15 to 4:15 p.m., only

Student Name: _____

School Name: _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. One solution of this system is

(1) $(-5,-2)$

(2) $(-1,-4)$

(3) $(1,4)$

(4) $(-2,-1)$

2 Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?

(1) The graph is always increasing.

(2) The graph is always decreasing.

(3) The graph passes through (1,0).

(4) The graph has an asymptote, $x = 0$.

3 For all values of x for which the expression is defined, $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$, in simplest form, is equivalent to

(1) 3

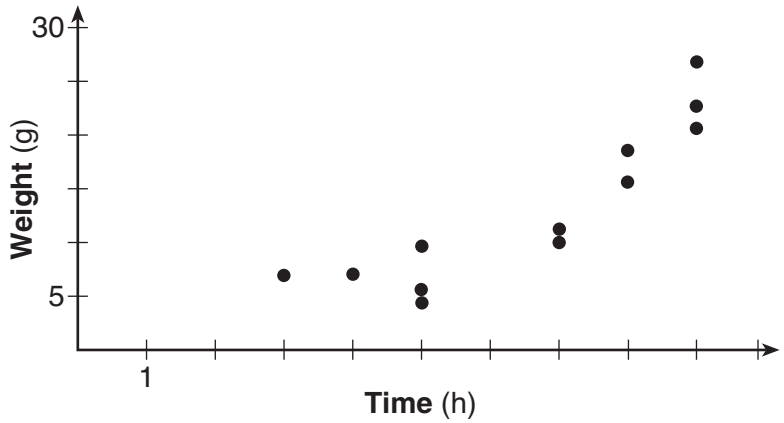
(2) $-\frac{17}{2}$

(3) $\frac{x+3}{x}$

(4) $\frac{x^2 - 9}{x(x-3)}$

Use this space for computations.

4 A scatterplot showing the weight, w, in grams, of each crystal after growing t hours is shown below.



The relationship between weight, w, and time, t, is best modeled by

(1) $w = 4^t + 5$

(2) $w = (1.4)^t + 2$

(3) $w = 5(2.1)^t$

(4) $w = 8(.75)^t$

5 Where i is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to

(1) $-3x^2$

(2) $-3x^2 - 18$

(3) $-3x^2 + 18xi$

(4) $-3x^2 - 6xi - 18$

6 Which function is even?

(1) $f(x) = \sin x$

(2) $f(x) = x^2 - 4$

(3) $f(x) = |x - 2| + 5$

(4) $f(x) = x^4 + 3x^3 + 4$

Use this space for computations.

7 The function $N(t) = 100e^{-0.023t}$ models the number of grams in a sample of cesium-137 that remain after t years. On which interval is the sample's average rate of decay the fastest?

(1) [1,10]

(2) [10,20]

(3) [15,25]

(4) [1,30]

8 Which expression can be rewritten as $(x + 7)(x - 1)$?

(1) $(x + 3)^2 - 16$

(2) $(x + 3)^2 - 10(x + 3) - 2(x + 3) + 20$

(3) $\frac{(x-1)(x^2-6x-7)}{(x+1)}$

(4) $\frac{(x+7)(x^2+4x+3)}{(x+3)}$

9 What is the solution set of the equation $\frac{2}{x} - \frac{3x}{x+3} = \frac{x}{x+3}$?

(1) $\{3\}$

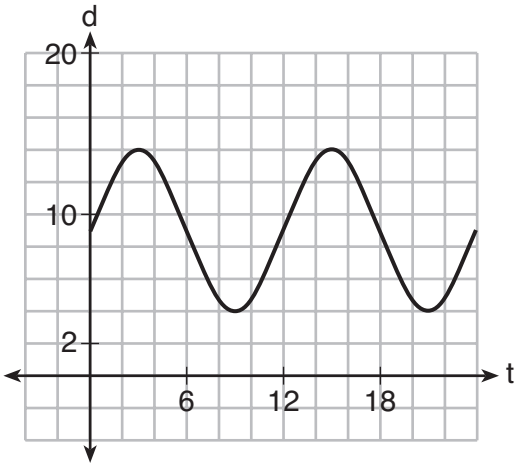
(2) $\left\{\frac{3}{2}\right\}$

(3) $\{-2,3\}$

(4) $\left\{-1,\frac{3}{2}\right\}$

Use this space for computations.

10 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.



If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

(1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$

(2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$

(3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$

(4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

11 On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

(1) 73%

(2) 36%

(3) 23%

(4) 12%

12 If $x - 1$ is a factor of $x^3 - kx^2 + 2x$, what is the value of k?

(1) 0

(2) 2

(3) 3

(4) -3

Use this space for computations.

13 The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

(1) $c(x) = 0.2x^2 - 100x + 300$

(2) $c(x) = 0.2x^2 + 100x + 300$

(3) $c(x) = -0.2x^2 + 100x - 300$

(4) $c(x) = -0.8x^2 + 400x - 300$

14 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

Town	Population	Annual Population Growth Rate
Jonesville	1240	6% increase
Williamstown	890	11% increase

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

(1) 7

(2) 20

(3) 68

(4) 125

15 What is the inverse of $f(x) = x^3 - 2$?

(1) $f^{-1}(x) = \sqrt[3]{x} + 2$

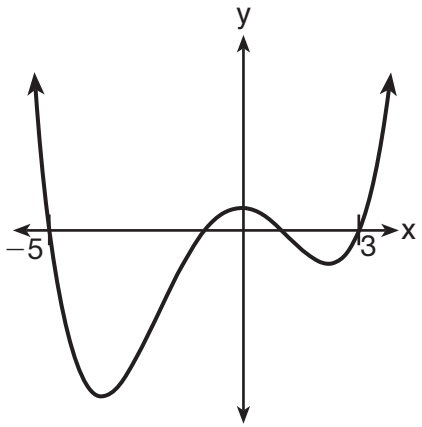
(2) $f^{-1}(x) = \pm\sqrt[3]{x} + 2$

(3) $f^{-1}(x) = \sqrt[3]{x + 2}$

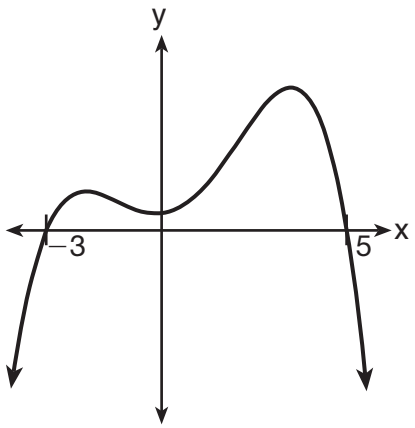
(4) $f^{-1}(x) = \pm\sqrt[3]{x + 2}$

Use this space for computations.

16 A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?



(1)



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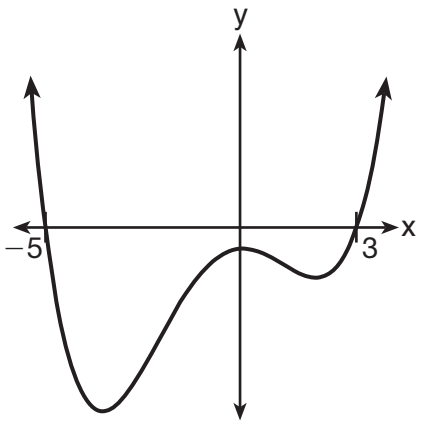
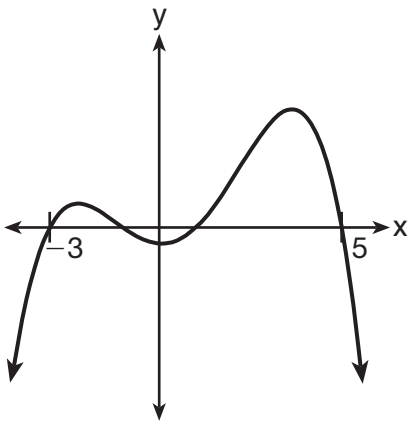


(2)



(4)

17 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

(1) 0.2257

(2) 0.2743

(3) 0.7257

(4) 0.7757

Use this space for computations.

18 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body d days after being introduced is $I = 100\left(\frac{1}{2}\right)^{\frac{d}{8}}$. When this equation is written in terms of the number e, the base of the natural logarithm, it is equivalent to $I = 100e^{kd}$. What is the approximate value of the constant, k?

(1) -0.087

(2) 0.087

(3) -11.542

(4) 11.542

19 The graph of $y = \log_2 x$ is translated to the right 1 unit and down 1 unit. The coordinates of the x-intercept of the translated graph are

(1) (0,0)

(2) (1,0)

(3) (2,0)

(4) (3,0)

20 For positive values of x, which expression is equivalent to $\sqrt{16x^2} \cdot x^{\frac{2}{3}} + \sqrt[3]{8x^5}$?

(1) $6\sqrt[5]{x^3}$

(2) $6\sqrt[3]{x^5}$

(3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$

(4) $4\sqrt{x^3} + 2\sqrt[5]{x^3}$

Use this space for computations.

21 Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y = 9$?

(1) $(y - 7)^2 = 8(x + 2)$

(2) $(y - 7)^2 = -8(x + 2)$

(3) $(x + 2)^2 = 8(y - 7)$

(4) $(x + 2)^2 = -8(y - 7)$

22 Given the following polynomials

$$x = (a + b + c)^2$$
$$y = a^2 + b^2 + c^2$$
$$z = ab + bc + ac$$

Which identity is true?

(1) $x = y - z$

(2) $x = y + z$

(3) $x = y - 2z$

(4) $x = y + 2z$

Use this space for computations.

23 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29{,}400(1.068)^t$, where t is time in years. Which expression is equivalent to $29{,}400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

(1) $29{,}400\left(1.068^{\frac{1}{365}}\right)^t$

(2) $29{,}400\left(\frac{1.068}{365}\right)^{365t}$

(3) $29{,}400\left(1 + \frac{0.068}{365}\right)^t$

(4) $29{,}400\left(1.068^{\frac{1}{365}}\right)^{365t}$

24 A manufacturing plant produces two different-sized containers of peanuts. One container weighs x ounces and the other weighs y pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

(1) $\frac{124}{16x + y}$

(2) $\frac{x + 16y}{124}$

(3) $\frac{124}{x + 16y}$

(4) $\frac{16x + y}{124}$

Part II

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Favorite Type of Program

	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshman	119	103	54

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

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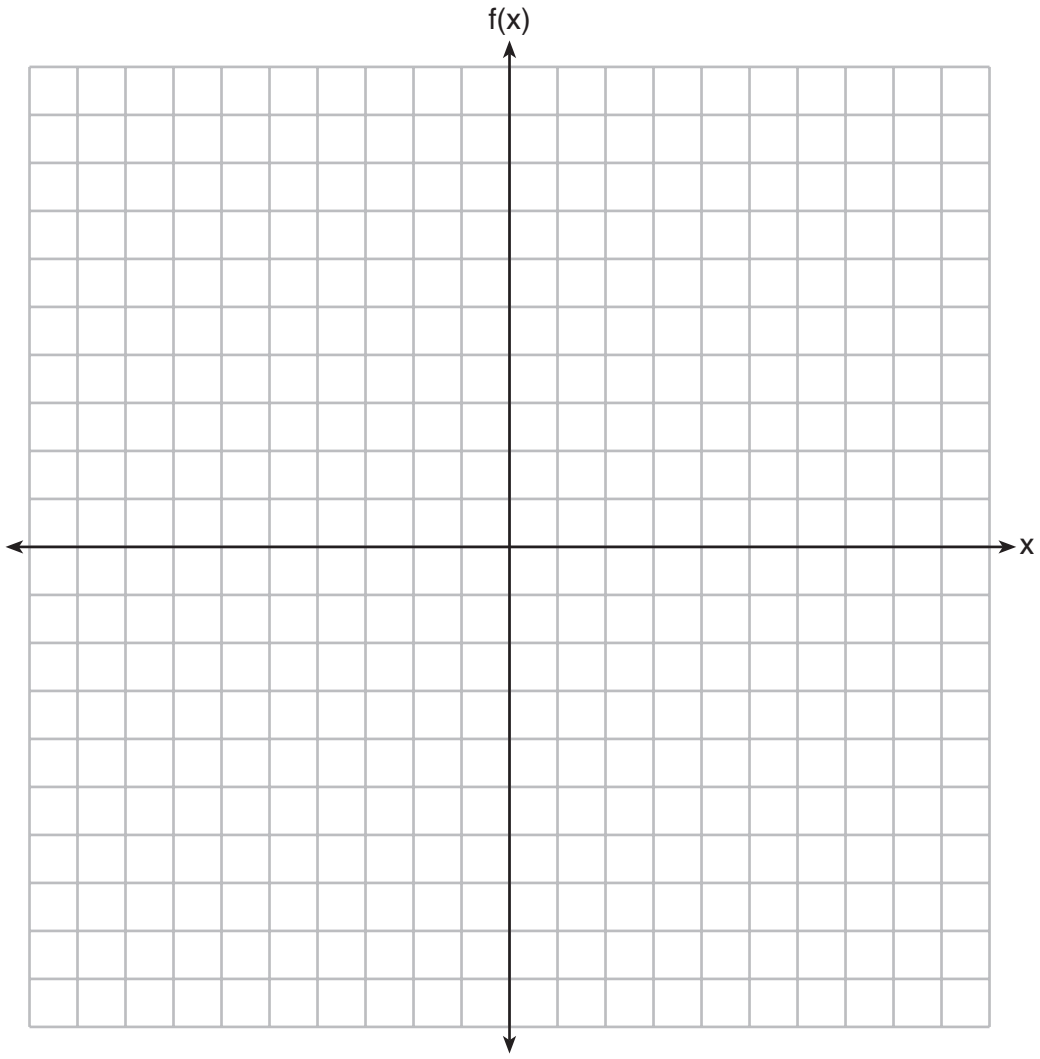

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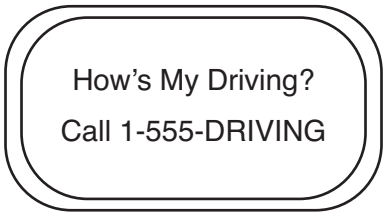

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Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$.

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$$a_1 = 3$$
$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence.

Can this sequence be represented using an explicit geometric formula? Justify your answer.

31 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = total number of monthly payments

32 An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve algebraically for all values of x:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

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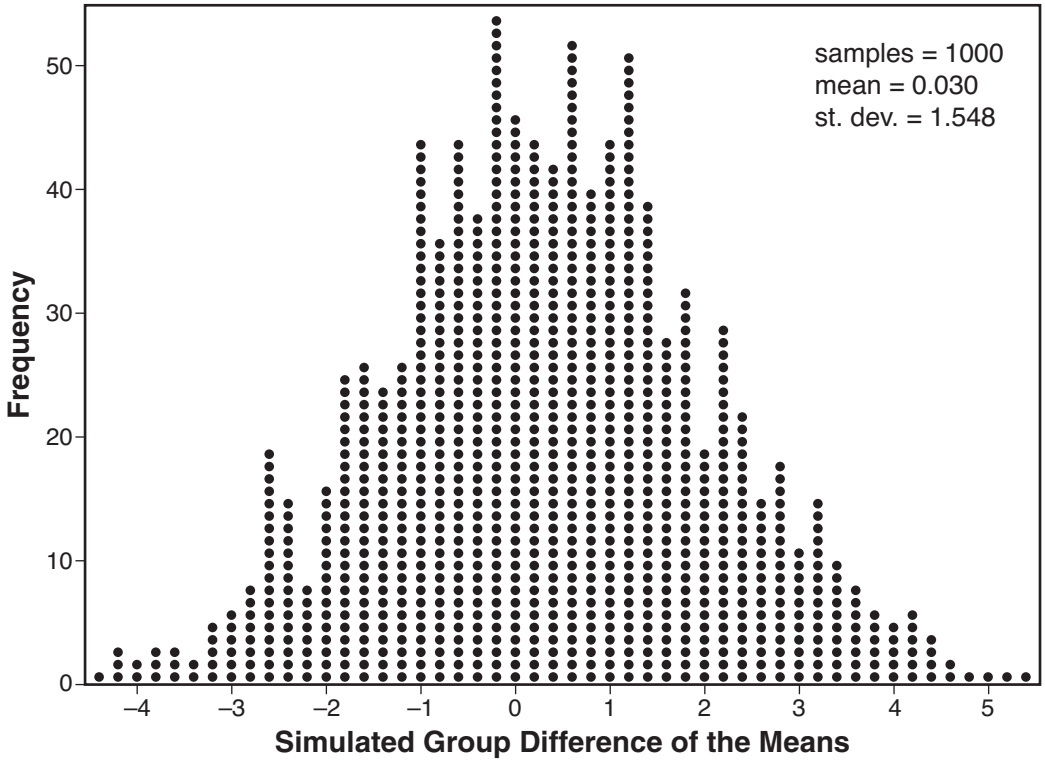
	Scented Paper	Unscented Paper
$\bar{x}$	23	18
s_x	2.898	2.408

Calculate the difference in means in the experimental test grades (scented – unscented).

A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Question 34 is continued on the next page.

Question 34 continued

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*.

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

35 Carla wants to start a college fund for her daughter Lila. She puts \$63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account.

Calculate algebraically the number of years it will take for the account to reach \$100,000, to the *nearest hundredth of a year*.

36 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$, where t represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \leq t \leq 2$.

At what value(s) of t, to the *nearest tenth of a second*, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

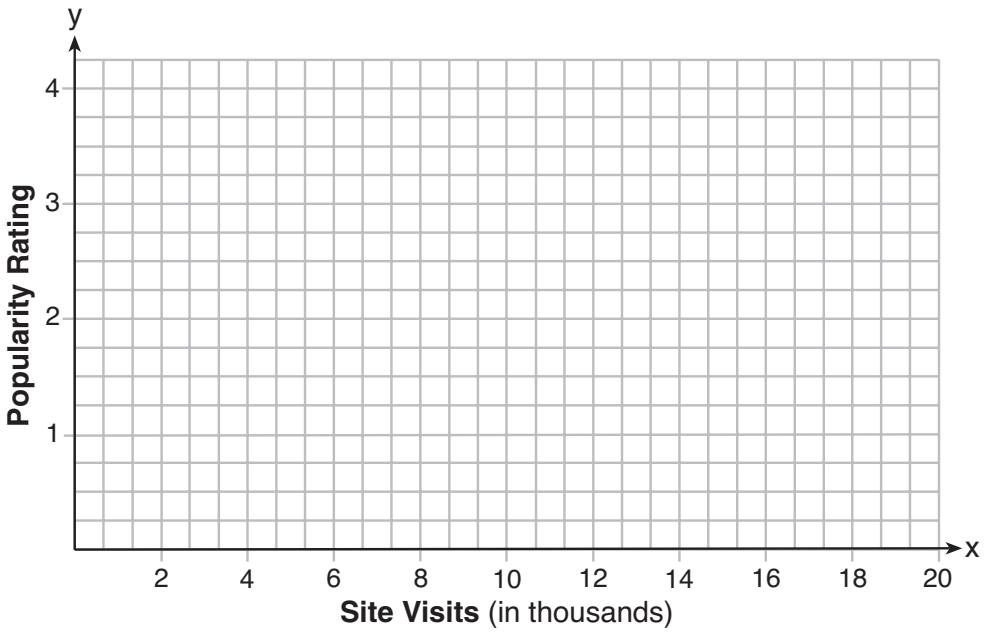
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*?

Graph $y = P(x)$ on the axes below.



An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graphs of the equations $y = x^2 + 4x - 1$ and $y + 3 = x$ are drawn on the same set of axes. One solution of this system is

(1) $(-5,-2)$

(2) $(-1,-4)$

(3) $(1,4)$

(4) $(-2,-1)$

$x^2+4x-1 = x-3$

$x^2+3x+2 = 0$

$(x+2)(x+1) = 0$

$x = -2, -1$

$y+3 = -1$

$y = -4$

2 Which statement is true about the graph of $f(x) = \left(\frac{1}{8}\right)^x$?

(1) The graph is always increasing.

(2) The graph is always decreasing.

(3) The graph passes through (1,0).

(4) The graph has an asymptote, $x = 0$.

exponential decay

3 For all values of x for which the expression is defined, $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$, in simplest form, is equivalent to

(1) 3

(2) $-\frac{17}{2}$

(3) $\frac{x+3}{x}$

(4) $\frac{x^2 - 9}{x(x-3)}$

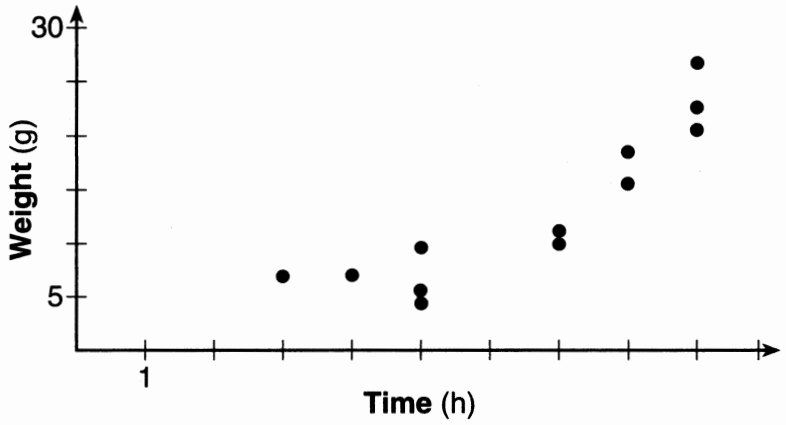
$\frac{x^2(x+2) - 9(x+2)}{x(x^2 - x - 6)}$

$\frac{(x^2-9)(x+2)}{x(x-3)(x+2)}$

$\frac{(x+3)(x-3)}{x(x-3)}$

Use this space for computations.

4 A scatterplot showing the weight, w, in grams, of each crystal after growing t hours is shown below.



The relationship between weight, w, and time, t, is best modeled by

(1) $w = 4^t + 5$
(2) $w = (1.4)^t + 2$
(3) $w = 5(2.1)^t$
(4) $w = 8(.75)^t$

5 Where i is the imaginary unit, the expression $(x + 3i)^2 - (2x - 3i)^2$ is equivalent to

(1) $-3x^2$
(2) $-3x^2 - 18$
(3) $-3x^2 + 18xi$
(4) $-3x^2 - 6xi - 18$

$x^2 + 6xi + 9i^2 - (4x^2 - 12xi + 9i^2)$
$-3x^2 + 18xi$

6 Which function is even?

(1) $f(x) = \sin x$
(2) $f(x) = x^2 - 4$
(3) $f(x) = |x - 2| + 5$
(4) $f(x) = x^4 + 3x^3 + 4$

$f(-x) = (-x)^2 - 4$
$= x^2 - 4$

Use this space for computations.

7 The function $N(t) = 100e^{-0.023t}$ models the number of grams in a sample of cesium-137 that remain after t years. On which interval is the sample's average rate of decay the fastest?

(1) [1,10]
(2) [10,20]
(3) [15,25]
(4) [1,30]

$\frac{N(10)-N(1)}{10-1} \approx -2.03$

$\frac{N(25)-N(15)}{25-15} \approx -1.46$

$\frac{N(20)-N(10)}{20-10} \approx -1.63$

$\frac{N(30)-N(1)}{30-1} \approx -1.64$

8 Which expression can be rewritten as $(x + 7)(x - 1)$?

(1) $(x + 3)^2 - 16$

(2) $(x + 3)^2 - 10(x + 3) - 2(x + 3) + 20$

(3) $\frac{(x-1)(x^2-6x-7)}{(x+1)}$

(4) $\frac{(x+7)(x^2+4x+3)}{(x+3)}$

x^2+6x-7

$x^2+6x+9-7-9$

$(x+3)^2-16$

9 What is the solution set of the equation $\frac{2}{x} - \frac{3x}{x+3} = \frac{x}{x+3}$?

(1) {3}
(2) $\left\{\frac{3}{2}\right\}$
(3) {−2,3}
(4) $\left\{-1, \frac{3}{2}\right\}$

$\frac{2}{x} = \frac{4x}{x+3}$

$4x^2 = 2x+6$

$4x^2-2x-6=0$

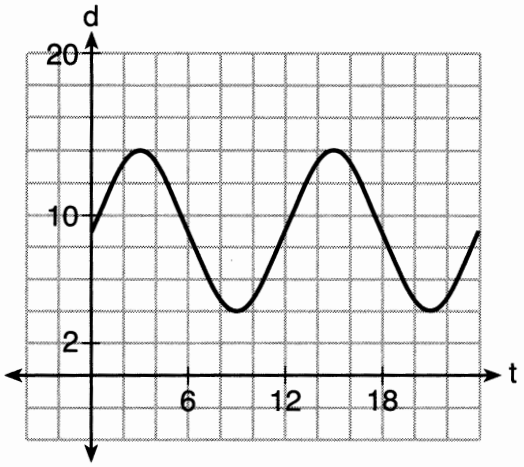
$2x^2-x-3=0$

$(2x-3)(x+1)=0$

$x = \frac{3}{2}, -1$

Use this space for computations.

10 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.



If the depth, d, is measured in feet and time, t, is measured in hours since midnight, what is an equation for the depth of the water at the marker?

(1) $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$

(2) $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$

(3) $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$

(4) $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

$a = \frac{14-4}{2} = 5$

$d = \frac{14+4}{2} = 9$

11 On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

(1) 73%

(2) 36%

(3) 23%

(4) 12%

$P(O \text{ and } Q) = P(O) \cdot P(Q)$
$= .48 \cdot .25$
$= .12$

12 If $x - 1$ is a factor of $x^3 - kx^2 + 2x$, what is the value of k?

(1) 0

(2) 2

(3) 3

(4) -3

$1^3 - k(1)^2 + 2(1) = 0$
$3 = k$

Use this space for computations.

13 The profit function, $p(x)$, for a company is the cost function, $c(x)$, subtracted from the revenue function, $r(x)$. The profit function for the Acme Corporation is $p(x) = -0.5x^2 + 250x - 300$ and the revenue function is $r(x) = -0.3x^2 + 150x$. The cost function for the Acme Corporation is

(1) $c(x) = 0.2x^2 - 100x + 300$

(2) $c(x) = 0.2x^2 + 100x + 300$

(3) $c(x) = -0.2x^2 + 100x - 300$

(4) $c(x) = -0.8x^2 + 400x - 300$

$p(x) = r(x) - c(x)$

$-0.5x^2 + 250x - 300 = -0.3x^2 + 150x - c(x)$

$c(x) = 0.2x^2 - 100x + 300$

14 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

Town	Population	Annual Population Growth Rate
Jonesville	1240	6% increase
Williamstown	890	11% increase

$1240(1.06)^x = 890(1.11)^x$

$x \approx 7$

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

(1) 7

(2) 20

(3) 68

(4) 125

15 What is the inverse of $f(x) = x^3 - 2$?

(1) $f^{-1}(x) = \sqrt[3]{x} + 2$

(2) $f^{-1}(x) = \pm\sqrt[3]{x} + 2$

(3) $f^{-1}(x) = \sqrt[3]{x + 2}$

(4) $f^{-1}(x) = \pm\sqrt[3]{x + 2}$

$y = x^3 - 2$

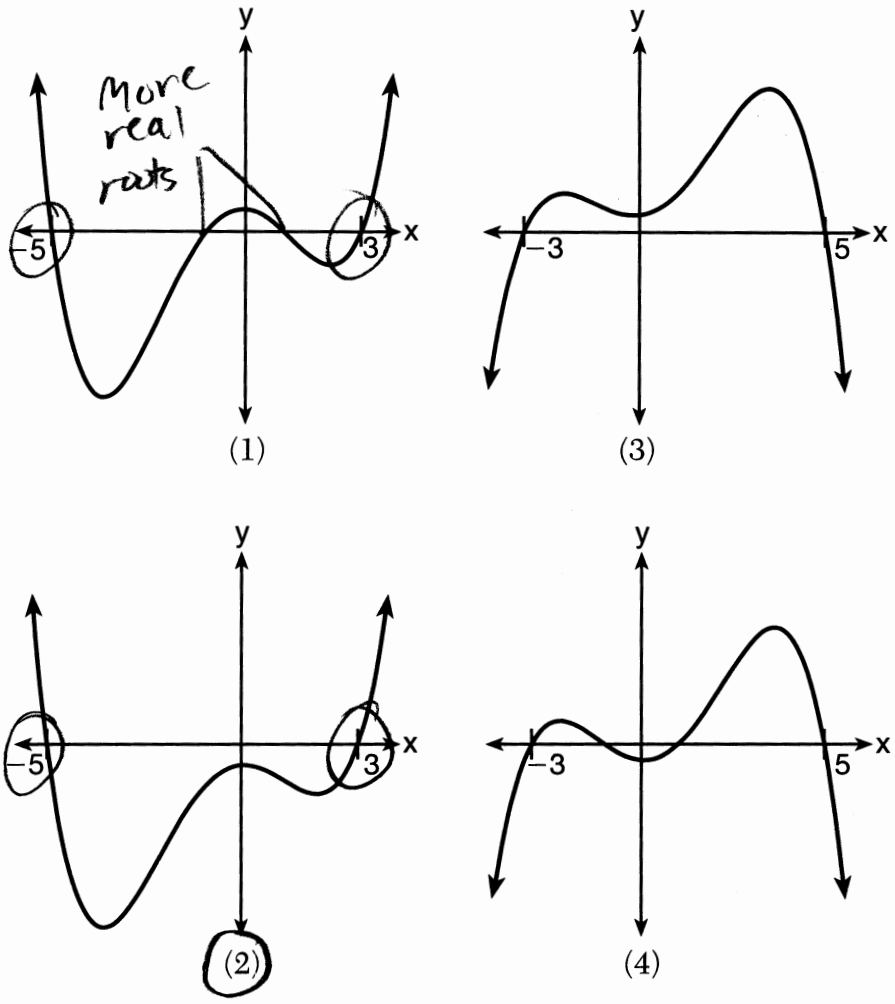
$x = y^3 - 2$

$x + 2 = y^3$

$\sqrt[3]{x + 2} = y$

Use this space for computations.

16 A 4th degree polynomial has zeros -5, 3, i, and $-i$. Which graph could represent the function defined by this polynomial?



17 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

(1) 0.2257
(2) 0.2743
(3) 0.7257
(4) 0.7757

18 The half-life of iodine-131 is 8 days. The percent of the isotope left in the body d days after being introduced is $I = 100\left(\frac{1}{2}\right)^{\frac{d}{8}}$. When this equation is written in terms of the number e, the base of the natural logarithm, it is equivalent to $I = 100e^{kd}$. What is the approximate value of the constant, k?

(1) -0.087
(2) 0.087
(3) -11.542
(4) 11.542

Use this space for computations.

$100\left(\frac{1}{2}\right)^{\frac{d}{8}} = 100e^{kd}$

$\ln\left(\frac{1}{2}\right)^{\frac{1}{8}} = \ln e^{k}$

$-0.087 \approx k$

19 The graph of $y = \log_2 x$ is translated to the right 1 unit and down 1 unit. The coordinates of the x-intercept of the translated graph are

(1) (0,0)
(2) (1,0)
(3) (2,0)
(4) (3,0)

$\log_2(x-1) - 1 = 0$

$\log_2(x-1) = 1$

$x - 1 = 2^1$

$x = 3$

20 For positive values of x, which expression is equivalent to $\sqrt{16x^2} \cdot x^{\frac{2}{3}} + \sqrt[3]{8x^5}$?

(1) $6\sqrt[5]{x^3}$
(2) $6\sqrt[3]{x^5}$
(3) $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$
(4) $4\sqrt{x^3} + 2\sqrt[5]{x^3}$

$4x \cdot x^{\frac{2}{3}} + 2x^{\frac{5}{3}}$

$4x^{\frac{5}{3}} + 2x^{\frac{5}{3}}$

$6x^{\frac{5}{3}}$

$6\sqrt[3]{x^5}$

Use this space for computations.

21 Which equation represents a parabola with a focus of $(-2,5)$ and a directrix of $y = 9$?

(1) $(y - 7)^2 = 8(x + 2)$

(2) $(y - 7)^2 = -8(x + 2)$

(3) $(x + 2)^2 = 8(y - 7)$

(4) $(x + 2)^2 = -8(y - 7)$

$(x+2)^2 = -8(y-7)$

$y = -\frac{1}{8}(x+2)^2 + 7 \quad p = 2$

vertex $(-2, 7)$

directrix $y = 7 + 2 = 9$

y value of focus $y = 7 - 2 = 5$

22 Given the following polynomials

$$x = (a + b + c)^2$$
$$y = a^2 + b^2 + c^2$$
$$z = ab + bc + ac$$

Which identity is true?

(1) $x = y - z$

(2) $x = y + z$

(3) $x = y - 2z$

(4) $x = y + 2z$

$x = y + 2x$

$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ac)$

$a^2+ab+ac+ab+b^2+bc+ac+ab+c^2 = a^2+b^2+c^2+2z$

$a^2+b^2+c^2+2ab+2bc+2ac = a^2+b^2+c^2+2ab+2bc+2ac$

Use this space for computations.

23 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function $D(t) = 29{,}400(1.068)^t$, where t is time in years. Which expression is equivalent to $29{,}400(1.068)^t$ and could be used by students to identify an approximate daily interest rate on their loans?

(1) $29{,}400\left(1.068^{\frac{1}{365}}\right)^t$

(2) $29{,}400\left(\frac{1.068}{365}\right)^{365t}$

(3) $29{,}400\left(1 + \frac{0.068}{365}\right)^t$

(4) $29{,}400\left(1.068^{\frac{1}{365}}\right)^{365t}$

24 A manufacturing plant produces two different-sized containers of peanuts. One container weighs x ounces and the other weighs y pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

1lb = 16 oz.

(1) $\frac{124}{16x + y}$

(2) $\frac{x + 16y}{124}$

(3) $\frac{124}{x + 16y}$

(4) $\frac{16x + y}{124}$

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Favorite Type of Program

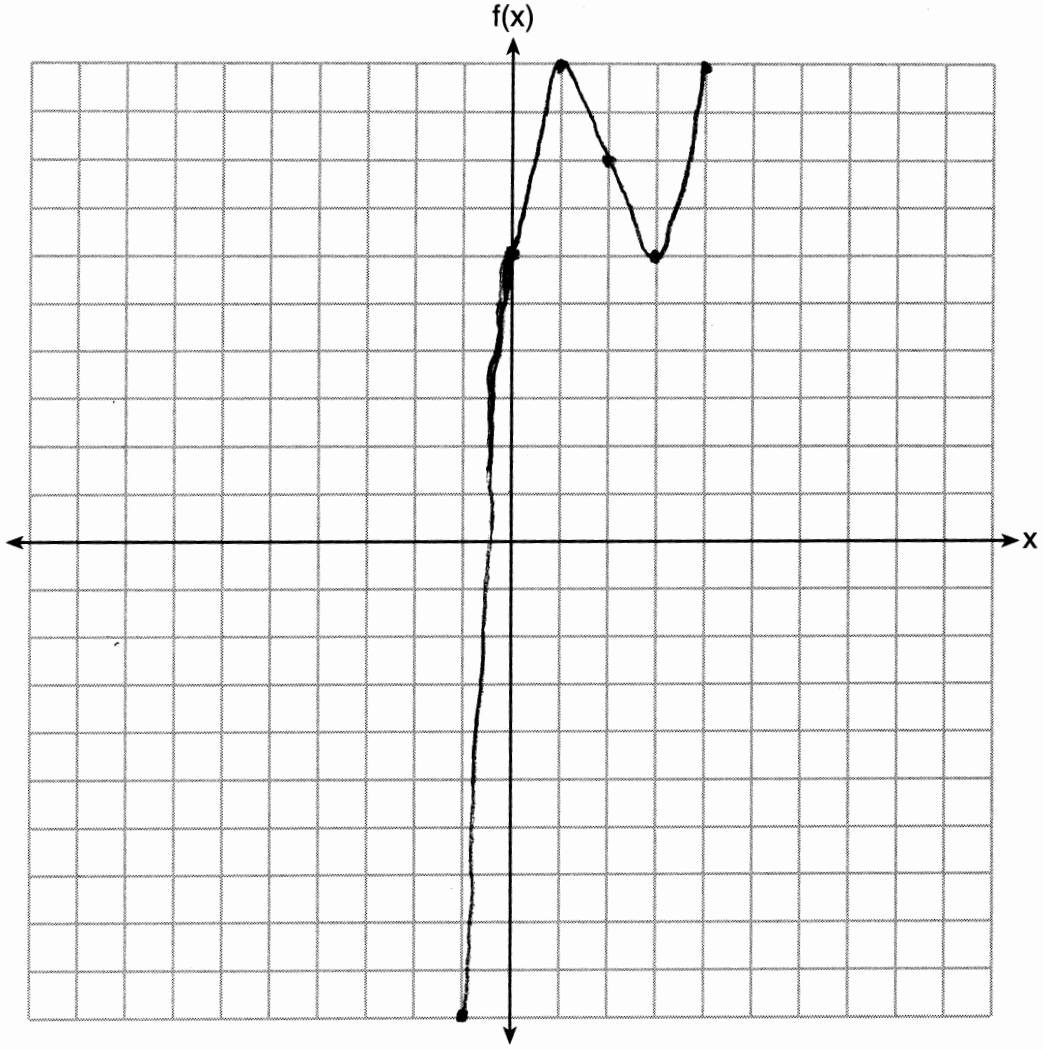
	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshman	119	103	54

213

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$\frac{103}{213}$

26 On the grid below, graph the function $f(x) = x^3 - 6x^2 + 9x + 6$ on the domain $-1 \leq x \leq 4$.



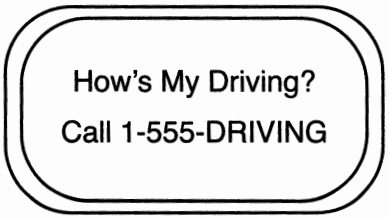
27 Solve the equation $2x^2 + 5x + 8 = 0$. Express the answer in $a + bi$ form.

$25 - 64 = -39$

$$X = \frac{-5 \pm \sqrt{5^2 - 4(2)(8)}}{2(2)}$$

$$= -\frac{5}{4} \pm \frac{i\sqrt{39}}{4}$$

28 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.



The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

Self selection is a cause of bias in this case because people with more free time or like surveys are more likely to respond.

29 Determine the quotient and remainder when $(6a^3 + 11a^2 - 4a - 9)$ is divided by $(3a - 2)$.

Express your answer in the form $q(a) + \frac{r(a)}{d(a)}$.

$2a^2 + 5a + 1 - \frac{5}{3a-2}$

$3a-2 \overline{)\ 6a^3 + 11a^2 - 4a - 9}$

$6a^3 - 4a^2$

$15a^2 - 4a$

$15a^2 - 10a$

$6a - 9$

$6a - 4$

-5

30 The recursive formula to describe a sequence is shown below.

$$a_1 = 3$$
$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence.

$a_1 = 3$

$a_2 = 7$

$a_3 = 15$

$a_4 = 31$

Can this sequence be represented using an explicit geometric formula? Justify your answer.

No, because there is no common ratio

$\frac{7}{3} \neq \frac{15}{7}$

31 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^{n}}{\left(1+\frac{r}{12}\right)^{n}-1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = total number of monthly payments

$$M = \frac{(152{,}500 - 15{,}250)\left(\frac{.036}{12}\right)\left(1+\frac{.036}{12}\right)^{360}}{\left(1+\frac{.036}{12}\right)^{360}-1} \approx 624$$

32 An angle, θ, is in standard position and its terminal side passes through the point $(2,-1)$. Find the *exact* value of $\sin\theta$.

$$\frac{-1}{\sqrt{2^2+(-1)^2}} : \frac{-1}{\sqrt{5}}$$

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve algebraically for all values of x:

$$\sqrt{6 - 2x} + x = 2(x + 15) - 9$$

$\sqrt{6-2x} + x = 2x + 30 - 9$

$\sqrt{6-2x} = x + 21$

$6 - 2x = x^2 + 42x + 441$

$x^2 + 44x + 435 = 0$

$(x + 29)(x + 15) = 0$

~~-29~~ -15

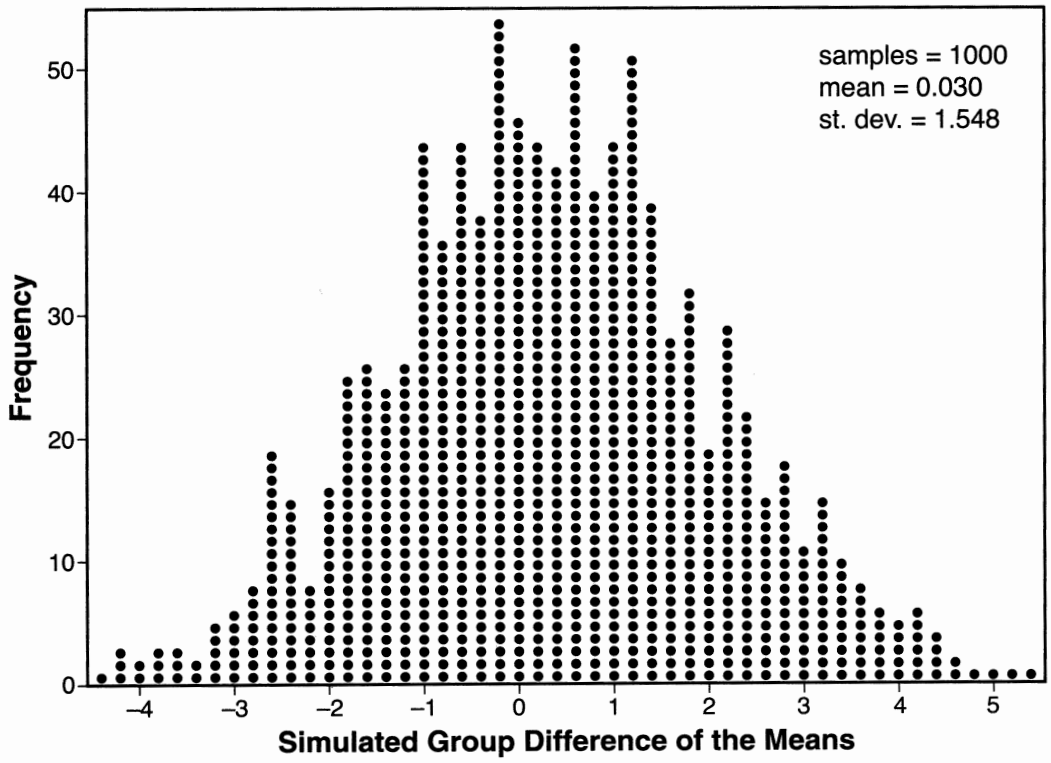
34 Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	Unscented Paper
$\bar{x}$	23	18
s_x	2.898	2.408

Calculate the difference in means in the experimental test grades (scented – unscented).

$23 - 18 = 5$

A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Question 34 is continued on the next page.

Question 34 continued

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*.

$\bar{x} \pm 2\sigma$

$-3.07 - 3.13$

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

Yes, a difference of 5 or more occurred 3 times out of 1000, which is statistically significant.

35 Carla wants to start a college fund for her daughter Lila. She puts \$63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function, $C(t)$, that represents the amount of money in the account t years after the account is opened, given that no more money is deposited into or withdrawn from the account.

$$C(t) = 63000\left(1 + \frac{.0255}{12}\right)^{12t}$$

Calculate algebraically the number of years it will take for the account to reach \$100,000, to the *nearest hundredth of a year*.

$$100\,000 = 63000\,(1.002125)^{12t}$$

$$\log \frac{100}{63} = 12t \log(1.002125)^{12t}$$

$$\frac{\log \frac{100}{63}}{12 \log(1.002125)} = t$$

$$18.14 \approx t$$

36 The height, $h(t)$ in cm, of a piston, is given by the equation $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$, where t represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval $1 \leq t \leq 2$.

$$\frac{h(2) - h(1)}{2 - 1} = -12$$

At what value(s) of t, to the *nearest tenth of a second*, does $h(t) = 0$ in the interval $1 \leq t \leq 5$? Justify your answer.

$h(t) = 0$ at $t \approx 2.2, 3.8$

Using graphing calculator to find where $h(t) = 0$

Part IV

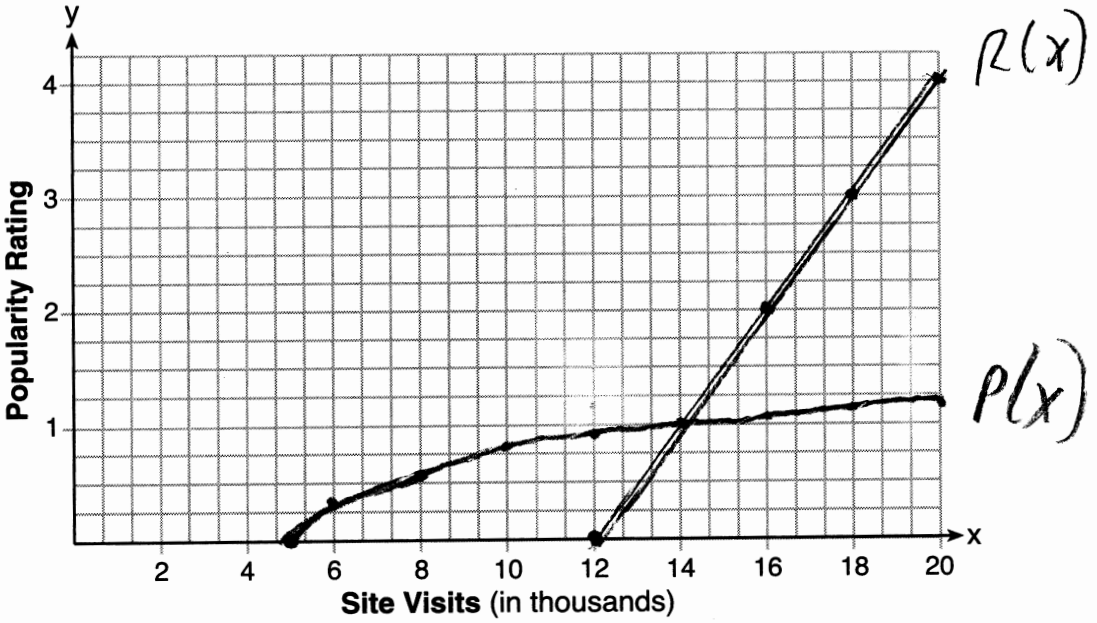
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is $P(x) = \log(x - 4)$, where x is the number of visits per week in thousands and $P(x)$ is the website's popularity rating.

According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the *nearest tenth*?

$P(16) = \log(16-4) \approx 1.1$

Graph $y = P(x)$ on the axes below.



An alternative rating model is represented by $R(x) = \frac{1}{2}x - 6$, where x is the number of visits per week in thousands. Graph $R(x)$ on the same set of axes. For what number of weekly visits will the two models provide the same rating?

14,000