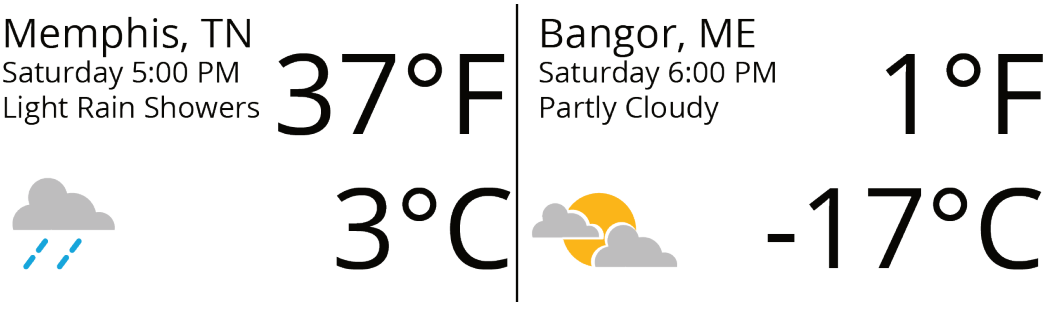
Illustrative Mathematics

Unit 7
Rational Numbers
Grade 6
Lessons 1-5

Lesson 1: Positive and Negative Numbers

1.1: Notice and Wonder: Memphis and Bangor



What do you notice? What do you wonder?

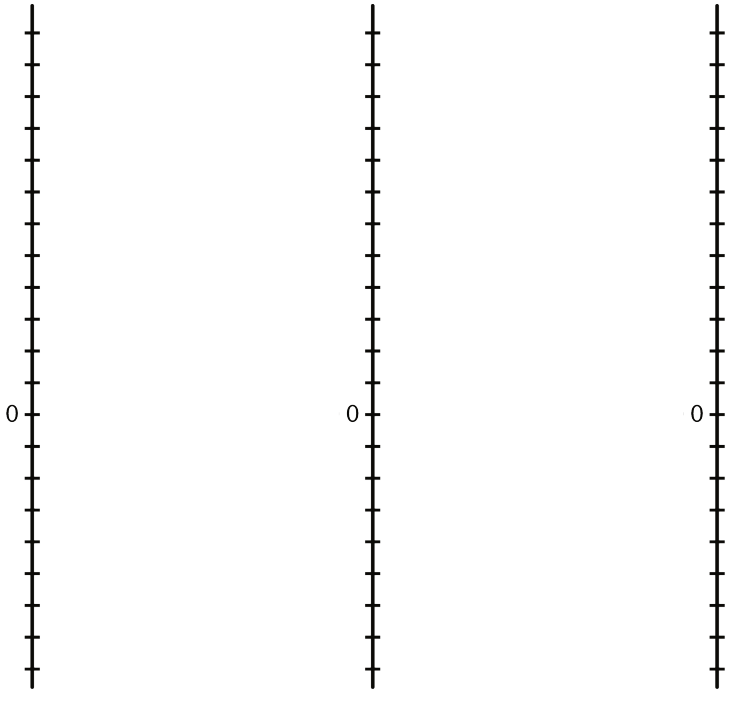
1.2: Above and Below Zero

1. Here are three situations involving changes in temperature and three number lines. Represent each change on a number line. Then, answer the question.

    a. At noon, the temperature was 5 degrees Celsius. By late afternoon, it has risen 6 degrees Celsius. What was the temperature late in the afternoon?

    b. The temperature was 8 degrees Celsius at midnight. By dawn, it has dropped 12 degrees Celsius. What was the temperature at dawn?

    c. Water freezes at 0 degrees Celsius, but the freezing temperature can be lowered by adding salt to the water. A student discovered that adding half a cup of salt to a gallon of water lowers its freezing temperature by 7 degrees Celsius. What is the freezing temperature of the gallon of salt water?



2. Discuss with a partner:

    a. How did each of you name the resulting temperature in each situation?

    b. What does it mean when the temperature is above 0? Below 0?

    c. Do numbers less than 0 make sense in other contexts? Give some specific examples to show how they do or do not make sense.

1.3: High Places, Low Places

1. Here is a table that shows elevations of various cities.

city	elevation (feet)
Harrisburg, PA	320
Bethell, IN	1,211
Denver, CO	5,280
Coachella, CA	-22
Death Valley, CA	-282
New York City, NY	33
Miami, FL	0

a. On the list of cities, which city has the second highest elevation?

b. How would you describe the elevation of Coachella, CA in relation to sea level?

c. How would you describe the elevation of Death Valley, CA in relation to sea level?

d. If you are standing on a beach right next to the ocean, what is your elevation?

e. How would you describe the elevation of Miami, FL?

f. A city has a higher elevation than Coachella, CA. Select all numbers that could represent the city's elevation. Be prepared to explain your reasoning.

- ■ -11 feet
- ■ -35 feet
- ■ 4 feet
- ■ -8 feet
- ■ 0 feet

2. Here are two tables that show the elevations of highest points on land and lowest points in the ocean. Distances are measured from sea level.

mountain	continent	elevation (meters)
Everest	Asia	8,848
Kilimanjaro	Africa	5,895
Denali	North America	6,168
Pikchu Pikchu	South America	5,664

trench	ocean	elevation (meters)
Mariana Trench	Pacific	-11,033
Puerto Rico Trench	Atlantic	-8,600
Tonga Trench	Pacific	-10,882
Sunda Trench	Indian	-7,725

a. Which point in the ocean is the lowest in the world? What is its elevation?

b. Which mountain is the highest in the world? What is its elevation?

c. If you plot the elevations of the mountains and trenches on a vertical number line, what would 0 represent? What would points above 0 represent? What about points below 0?

d. Which is farther from sea level: the deepest point in the ocean, or the top of the highest mountain in the world? Explain.

Are you ready for more?

A spider spins a web in the following way:

- It starts at sea level.
- It moves up one inch in the first minute.
- It moves down two inches in the second minute.
- It moves up three inches in the third minute.
- It moves down four inches in the fourth minute.

Assuming that the pattern continues, what will the spider's elevation be after an hour has passed?

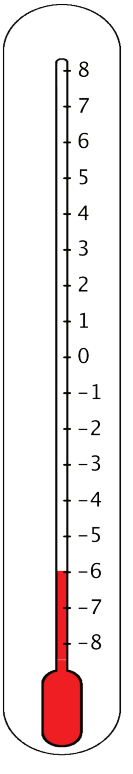
Lesson 1 Summary

Positive numbers are numbers that are greater than 0. **Negative numbers** are numbers that are less than zero. The meaning of a negative number in a context depends on the meaning of zero in that context.

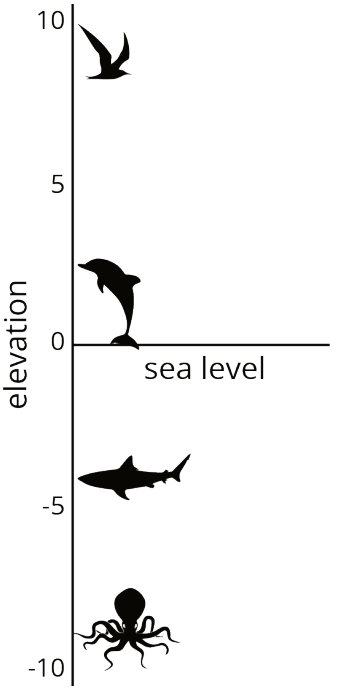
For example, if we measure temperatures in degrees Celsius, then 0 degrees Celsius corresponds to the temperature at which water freezes.

In this context, positive temperatures are warmer than the freezing point and negative temperatures are colder than the freezing point. A temperature of -6 degrees Celsius means that it is 6 degrees away from 0 and it is less than 0. This thermometer shows a temperature of -6 degrees Celsius.

If the temperature rises a few degrees and gets very close to 0 degrees without reaching it, the temperature is still a negative number.



Another example is elevation, which is a distance above or below sea level. An elevation of 0 refers to the sea level. Positive elevations are higher than sea level, and negative elevations are lower than sea level.



NAME DATE PERIOD

Cool Down

Lesson 1: Positive and Negative Numbers

Cool Down: Agree or Disagree?

State whether you agree with each of the following statements. Explain your reasoning.

1. A temperature of 35 degrees Fahrenheit is as cold as a temperature of -35 degrees Fahrenheit.
2. A city that has an elevation of 15 meters is closer to sea level than a city that has an elevation of -10 meters.
3. A city that has an elevation of -17 meters is closer to sea level than a city that has an elevation of -40 meters.

Unit 7 Lesson 1 Cumulative Practice Problems

1. a. Is a temperature of -11 degrees warmer or colder than a temperature of -15 degrees?

   b. Is an elevation of -10 feet closer or farther from the surface of the ocean than an elevation of -8 feet?

   c. It was 8 degrees at nightfall. The temperature dropped 10 degrees by midnight. What was the temperature at midnight?

   d. A diver is 25 feet below sea level. After he swims up 15 feet toward the surface, what is his elevation?

2. a. A whale is at the surface of the ocean to breathe. What is the whale's elevation?

   b. The whale swims down 300 feet to feed. What is the whale's elevation now?

   c. The whale swims down 150 more feet more. What is the whale's elevation now?

   d. Plot each of the three elevations as a point on a vertical number line. Label each point with its numeric value.

3. Explain how to calculate a number that is equal to $\frac{2.1}{1.5}$.

(From Unit 6, Lesson 5.)

4. Write an equation to represent each situation and then solve the equation.

   a. Andre drinks 15 ounces of water, which is $\frac{3}{5}$ of a bottle. How much does the bottle hold? Use x for the number of ounces of water the bottle holds.

   b. A bottle holds 15 ounces of water. Jada drank 8.5 ounces of water. How many ounces of water are left in the bottle? Use y for the number of ounces of water left in the bottle.

   c. A bottle holds z ounces of water. A second bottle holds 16 ounces, which is $\frac{8}{5}$ times as much water. How much does the first bottle hold?

   (From Unit 6, Lesson 4.)

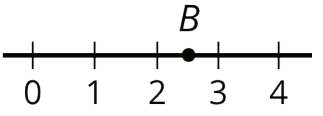
5. A rectangle has an area of 24 square units and a side length of $2\frac{3}{4}$ units. Find the other side length of the rectangle. Show your reasoning.

   (From Unit 4, Lesson 13.)

Lesson 2: Points on the Number Line

2.1: A Point on the Number Line

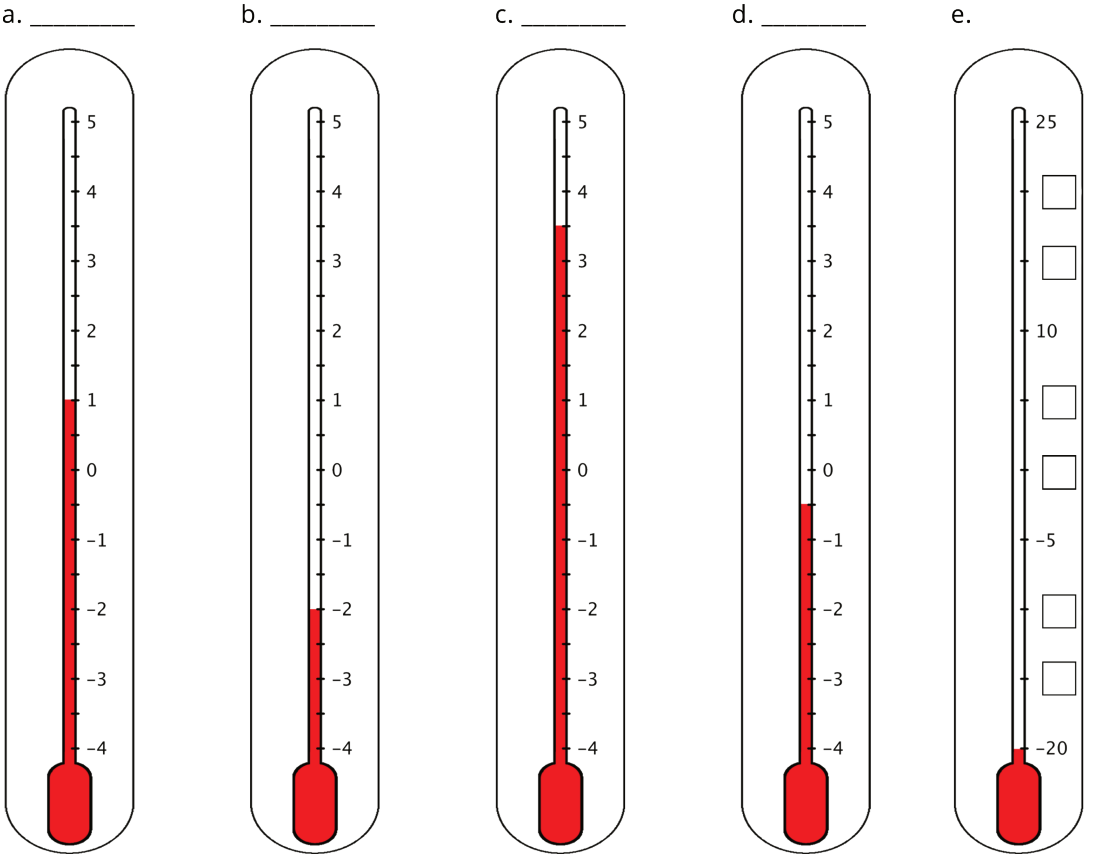
Which of the following numbers could be B?



2.5 $\frac{2}{5}$ $\frac{5}{2}$ $\frac{25}{10}$ 2.49

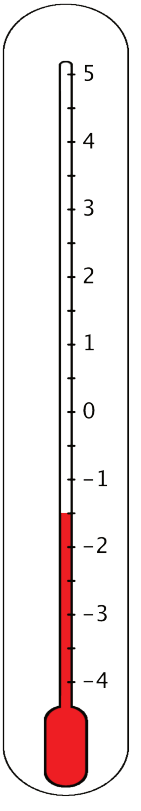
2.2: What's the Temperature?

1. Here are five thermometers. The first four thermometers show temperatures in Celsius. Write the temperatures in the blanks.



The last thermometer is missing some numbers. Write them in the boxes.

2. Elena says that the thermometer shown here reads $-2.5^\circ C$ because the line of the liquid is above $-2^\circ C$. Jada says that it is $-1.5^\circ C$. Do you agree with either one of them? Explain your reasoning.



3. One morning, the temperature in Phoenix, Arizona, was $8^\circ C$ and the temperature in Portland, Maine, was $12^\circ C$ cooler. What was the temperature in Portland?

2.3: Folded Number Lines

Your teacher will give you a sheet of tracing paper on which to draw a number line.

1. Follow the steps to make your own number line.
    - Use a straightedge or a ruler to draw a horizontal line. Mark the middle point of the line and label it 0.
    - To the right of 0, draw tick marks that are 1 centimeter apart. Label the tick marks 1, 2, 3. . . 10. This represents the positive side of your number line.
    - Fold your paper so that a vertical crease goes through 0 and the two sides of the number line match up perfectly.
    - Use the fold to help you trace the tick marks that you already drew onto the opposite side of the number line. Unfold and label the tick marks -1, -2, -3. . . -10. This represents the negative side of your number line.

2. Use your number line to answer these questions:

   a. Which number is the same distance away from zero as is the number 4?

   b. Which number is the same distance away from zero as is the number -7?

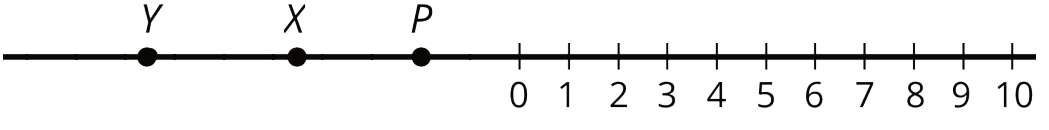
   c. Two numbers that are the same distance from zero on the number line are called **opposites**. Find another pair of opposites on the number line.

   d. Determine how far away the number 5 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number 5.

   e. Determine how far away the number -2 is from 0. Then, choose a positive number and a negative number that is each farther away from zero than is the number -2.

   Pause here so your teacher can review your work.

3. Here is a number line with some points labeled with letters. Determine the location of points P, X, and Y.



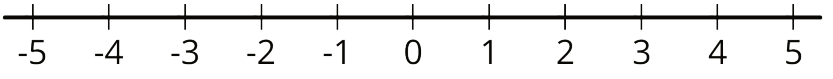
   If you get stuck, trace the number line and points onto a sheet of tracing paper, fold it so that a vertical crease goes through 0, and use the folded number line to help you find the unknown values.

Are you ready for more?

At noon, the temperatures in Portland, Maine, and Phoenix, Arizona, had opposite values. The temperature in Portland was 18°C lower than in Phoenix. What was the temperature in each city? Explain your reasoning.

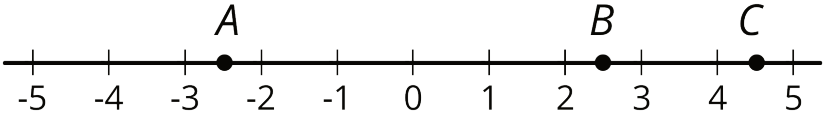
Lesson 2 Summary

Here is a number line labeled with positive and negative numbers. The number 4 is positive, so its location is 4 units to the right of 0 on the number line. The number -1.1 is negative, so its location is 1.1 units to the left of 0 on the number line.



We say that the *opposite* of 8.3 is -8.3, and that the *opposite* of $\frac{-3}{2}$ is $\frac{3}{2}$. Any pair of numbers that are equally far from 0 are called **opposites**.

Points A and B are opposites because they are both 2.5 units away from 0, even though A is to the left of 0 and B is to the right of 0.



A positive number has a negative number for its opposite. A negative number has a positive number for its opposite. The opposite of 0 is itself.

You have worked with positive numbers for many years. All of the positive numbers you have seen—whole and non-whole numbers—can be thought of as fractions and can be located on a the number line.

To locate a non-whole number on a number line, we can divide the distance between two whole numbers into fractional parts and then count the number of parts. For example, 2.7 can be written as $2\frac{7}{10}$. The segment between 2 and 3 can be partitioned into 10 equal parts or 10 tenths. From 2, we can count 7 of the tenths to locate 2.7 on the number line.

All of the fractions and their opposites are what we call **rational numbers**. For example, 4, -1.1, 8.3, -8.3, $\frac{-3}{2}$, and $\frac{3}{2}$ are all rational numbers.

NAME DATE PERIOD

Cool Down

Lesson 2: Points on the Number Line

Cool Down: Positive, Negative, and Opposite

1. Put these numbers in order, from least to greatest. If you get stuck, consider using the number line.

   3.5

   -1

   4.8

   -1.5

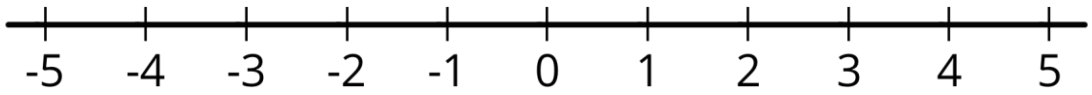
   -0.5

   -4.2

   0.5

   -2.1

   -3.5



2. Write two numbers that are opposites and each more than 6 units away from 0.

Unit 7 Lesson 2 Cumulative Practice Problems

1. For each number, name its opposite.

   a. -5

   b. 28

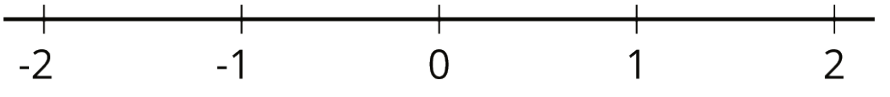
   c. -10.4

   a. 0.875

   b. 0

   c. -8,003

2. Plot the numbers -1.5, $\frac{3}{2}$, $-\frac{3}{2}$, and $-\frac{4}{3}$ on the number line. Label each point with its numeric value.



3. Plot these points on a number line.

   - -1.5
   - the opposite of -2
   - the opposite of 0.5
   - -2

4. a. Represent each of these temperatures in degrees Fahrenheit with a positive or negative number.

      - 5 degrees above zero
      - 3 degrees below zero
      - 6 degrees above zero
      - $2\frac{3}{4}$ degrees below zero

   b. Order the temperatures above from the coldest to the warmest.

(From Unit 7, Lesson 1.)

5. Solve each equation.

a. $8x = \frac{2}{3}$

b. $1\frac{1}{2} = 2x$

c. $5x = \frac{2}{7}$

d. $\frac{1}{4}x = 5$

e. $\frac{1}{5} = \frac{2}{3}x$

(From Unit 6, Lesson 5.)

6. Write the solution to each equation as a fraction and as a decimal.

a. $2x = 3$

b. $5y = 3$

c. $0.3z = 0.009$

(From Unit 6, Lesson 5.)

7. There are 15.24 centimeters in 6 inches.

a. How many centimeters are in 1 foot?

b. How many centimeters are in 1 yard?

(From Unit 3, Lesson 4.)

Lesson 3: Comparing Positive and Negative Numbers

3.1: Which One Doesn't Belong: Inequalities

Which inequality doesn't belong?

- $\frac{5}{4} < 2$
- $8.5 > 0.95$
- $8.5 < 7$
- $10.00 < 100$

3.2: Comparing Temperatures

Here are the low temperatures, in degrees Celsius, for a week in Anchorage, Alaska.

day	Mon	Tues	Weds	Thurs	Fri	Sat	Sun
temperature	5	-1	-5.5	-2	3	4	0

1. Plot the temperatures on a number line. Which day of the week had the lowest low temperature?

2. The lowest temperature ever recorded in the United States was -62 degrees Celsius, in Prospect Creek Camp, Alaska. The average temperature on Mars is about -55 degrees Celsius.

   a. Which is warmer, the coldest temperature recorded in the USA, or the average temperature on Mars? Explain how you know.

   b. Write an inequality to show your answer.

3. On a winter day the low temperature in Anchorage, Alaska, was -21 degrees Celsius and the low temperature in Minneapolis, Minnesota, was -14 degrees Celsius.

   Jada said, "I know that 14 is less than 21, so -14 is also less than -21. This means that it was colder in Minneapolis than in Anchorage."

   Do you agree? Explain your reasoning.

Are you ready for more?

Another temperature scale frequently used in science is the *Kelvin scale*. In this scale, 0 is the lowest possible temperature of anything in the universe, and it is -273.15 degrees in the Celsius scale. Each $1\ \text{K}$ is the same as 1°C, so $10\ \text{K}$ is the same as -263.15°C.

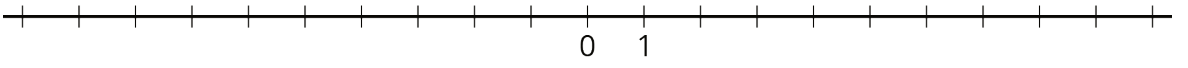
1. Water boils at 100°C. What is this temperature in K?

2. Ammonia boils at -35.5°C. What is the boiling point of ammonia in K?

3. Explain why only positive numbers (and 0) are needed to record temperature in K.

3.3: Rational Numbers on a Number Line

1. Plot the numbers -2, 4, -7, and 10 on the number line. Label each point with its numeric value.



2. Decide whether each inequality statement is true or false. Be prepared to explain your reasoning.

   a. $-2 < 4$

   b. $-2 < -7$

   c. $4 > -7$

   d. $-7 > 10$

3. Andre says that $\frac{1}{4}$ is less than $-\frac{3}{4}$ because, of the two numbers, $\frac{1}{4}$ is closer to 0. Do you agree? Explain your reasoning.

4. Answer each question. Be prepared to explain how you know.

   a. Which number is greater: $\frac{1}{4}$ or $\frac{5}{4}$?

   b. Which is farther from 0: $\frac{1}{4}$ or $\frac{5}{4}$?

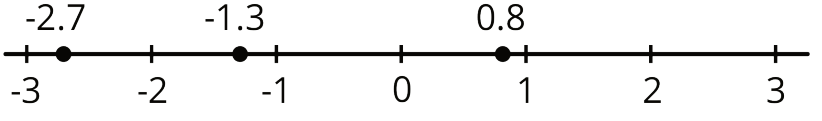
   c. Which number is greater: $-\frac{3}{4}$ or $\frac{5}{8}$?

   d. Which is farther from 0: $-\frac{3}{4}$ or $\frac{5}{8}$?

e. Is the number that is farther from 0 always the greater number? Explain your reasoning.

Lesson 3 Summary

We use the words *greater than* and *less than* to compare numbers on the number line. For example, the numbers -2.7, 0.8, and -1.3, are shown on the number line.



Because -2.7 is to the left of -1.3, we say that -2.7 is less than -1.3. We write:

$$-2.7 < -1.3$$

In general, any number that is to the left of a number n is less than n.

We can see that -1.3 is greater than -2.7 because -1.3 is to the right of -2.7. We write:

$$-1.3 > -2.7$$

In general, any number that is to the right of a number n is greater than n

We can also see that $0.8 > -1.3$ and $0.8 > -2.7$. In general, any positive number is greater than any negative number.

NAME DATE PERIOD

Cool Down

Lesson 3: Comparing Positive and Negative Numbers

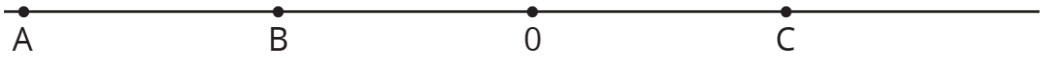
Cool Down: Making More Comparisons

1. The elevation of Death Valley, California, is -282 feet. The elevation of Tallahassee, Florida, is 203 feet. The elevation of Westmorland, California, is -157 feet.

   a. Compare the elevations of Death Valley and Tallahassee using $<$ or $>$.

   b. Compare the elevations of Death Valley and Westmorland.

2. Here are the points A, B, C, and 0 plotted on a number line.



   The points B and C are opposites. Decide whether each of the following statements is true.

   a. A is greater than B.

   b. A is farther from 0 than C.

   c. A is less than C.

   d. B and C are equally far away from 0.

   e. B and C are equal.

Unit 7 Lesson 3 Cumulative Practice Problems

1. Decide whether each inequality statement is true or false. Explain your reasoning.

   a. $-5 > 2$

   b. $3 > -8$

   c. $-12 > -15$

   d. $-12.5 > -12$

2. Here is a true statement: $-8.7 < -8.4$. Select **all** of the statements that are equivalent to $-8.7 < -8.4$.

   A. -8.7 is further to the right on the number line than -8.4.

   B. -8.7 is further to the left on the number line than -8.4.

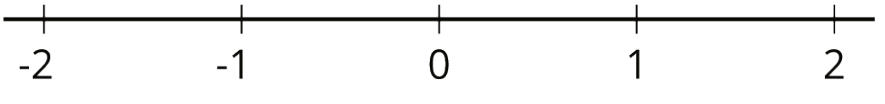
   C. -8.7 is less than -8.4.

   D. -8.7 is greater than -8.4.

   E. -8.4 is less than -8.7.

   F. -8.4 is greater than -8.7.

3. Plot each of the following numbers on the number line. Label each point with its numeric value. 0.4, -1.5, $-1\frac{7}{10}$, $-\frac{11}{10}$



(From Unit 7, Lesson 2.)

4. The table shows five states and the lowest point in each state.

   Put the states in order by their lowest elevation, from least to greatest.

state	lowest elevation (feet)
California	-282
Colorado	3350
Louisiana	-8
New Mexico	2842
Wyoming	3099

(From Unit 7, Lesson 4.)

5. Each lap around the track is 400 meters.

   a. How many meters does someone run if they run:

   2 laps? 5 laps? x laps?

   b. If Noah ran 14 laps, how many meters did he run?

   c. If Noah ran 7,600 meters, how many laps did he run?

(From Unit 6, Lesson 6.)

6. A stadium can seat 16,000 people at full capacity.

   a. If there are 13,920 people in the stadium, what percentage of the capacity is filled? Explain or show your reasoning.

   b. What percentage of the capacity is not filled?

(From Unit 3, Lesson 16.)

Lesson 4: Ordering Rational Numbers

4.1: How Do They Compare?

Use the symbols >, <, or = to compare each pair of numbers. Be prepared to explain your reasoning.

- 12 _____ 19
- 212 _____ 190
- 15 _____ 1.5
- 9.02 _____ 9.2
- 6.050 _____ 6.05
- 0.4 _____ $\frac{9}{40}$
- $\frac{19}{24}$ _____ $\frac{19}{21}$
- $\frac{16}{17}$ _____ $\frac{11}{12}$

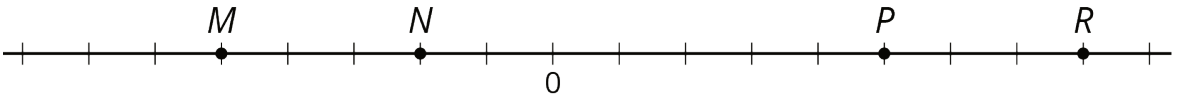
4.2: Ordering Rational Number Cards

Your teacher will give you a set of number cards. Order them from least to greatest.

Your teacher will give you a second set of number cards. Add these to the correct places in the ordered set.

4.3: Comparing Points on A Line

1.



Use each of the following terms at least once to describe or compare the values of points M, N, P, R.

- greater than
- less than
- opposite of (or opposites)
- negative number

2. Tell what the value of each point would be if:

a. P is $2\frac{1}{2}$

b. N is -0.4

c. R is 200

d. M is -15

Are you ready for more?

The list of fractions between 0 and 1 with denominators between 1 and 3 looks like this:

$$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}$$

We can put them in order like this: $\frac{0}{1} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{1}{1}$

Now let's expand the list to include fractions with denominators of 4. We won't include $\frac{2}{4}$, because $\frac{1}{2}$ is already on the list.

$$\frac{0}{1} < \frac{1}{4} < \frac{1}{3} < \frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{1}{1}$$

1. Expand the list again to include fractions that have denominators of 5.

2. Expand the list you made to include fractions have have denominators of 6.

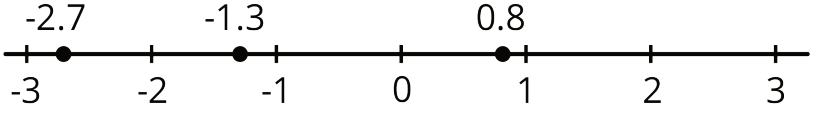
3. When you add a new fraction to the list, you put it in between two "neighbors." Go back and look at your work. Do you see a relationship between a new fraction and its two neighbors?

Lesson 4 Summary

To order rational numbers from least to greatest, we list them in the order they appear on the number line from left to right. For example, we can see that the numbers

-2.7, -1.3, 0.8

are listed from least to greatest because of the order they appear on the number line.



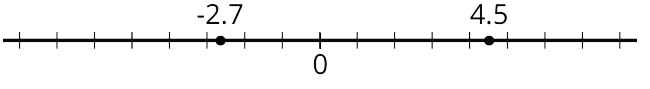
Lesson 4: Ordering Rational Numbers

Cool Down: Getting Them in Order

1. Place these numbers in order from least to greatest:

$\frac{16}{5}$ -3 6 3.1 -2.5 $\frac{1}{4}$ $-\frac{3}{4}$ $-\frac{3}{8}$

2. Write a sentence to compare the two points shown on the number line.



Unit 7 Lesson 4 Cumulative Practice Problems

1. Select **all** of the numbers that are *greater than* -5.

   A. 1.3

   B. -6

   C. -12

   D. $\frac{1}{7}$

   E. -1

   F. -4

2. Order these numbers from least to greatest: $\frac{1}{2}$, 0, 1, $-1\frac{1}{2}$, $-\frac{1}{2}$, -1

3. Here are the boiling points of certain elements in degrees Celsius:

   - Argon: -185.8
   - Chlorine: -34
   - Fluorine: -188.1
   - Hydrogen: -252.87
   - Krypton: -153.2

   List the elements from least to greatest boiling points.

4. Explain why zero is considered its own opposite.

   (From Unit 7, Lesson 2.)

5. Explain how to make these calculations mentally.

   a. $99 + 54$

   b. $244 - 99$

   c. $99 \cdot 6$

   d. $99 \cdot 15$

   (From Unit 6, Lesson 9.)

6. Find the quotients.

   a. $\frac{1}{2} \div 2$

   b. $2 \div 2$

   c. $\frac{1}{2} \div \frac{1}{2}$

   d. $\frac{38}{79} \div \frac{38}{79}$

   (From Unit 4, Lesson 11.)

7. Over several months, the weight of a baby measured in pounds doubles. Does its weight measured in kilograms also double? Explain.

   (From Unit 3, Lesson 4.)

Lesson 5: Using Negative Numbers to Make Sense of Contexts

5.1: Notice and Wonder: It Comes and Goes

activity	amount
do my chores	30.00
babysit my cousin	45.00
buy my lunch	-10.80
get my allowance	15.00
buy a shirt	-18.69
pet my dog	0.00

What do you notice? What do you wonder?

5.2: The Concession Stand

The manager of the concession stand keeps records of all of the supplies she buys and all of the items she sells. The table shows some of her records for Tuesday.

item	quantity	value in dollars
doughnuts	-58	37.70
straws	3,000	-10.35
hot dogs	-39	48.75
pizza	13	-116.87
apples	-40	14.00
french fries	-88	132.00

1. Which items did she sell? Explain your reasoning.

2. How can we interpret -58 in this situation?

3. How can we interpret -10.35 in this situation?

4. On which item did she spend the most amount of money? Explain your reasoning.

5.3: Drinks for Sale

A vending machine in an office building sells bottled beverages. The machine keeps track of all changes in the number of bottles from sales and from machine refills and maintenance. This record shows the changes for every 5-minute period over one hour.

1. What might a positive number mean in this context? What about a negative number?

2. What would a "0" in the second column mean in this context?

3. Which numbers—positive or negative—result in fewer bottles in the machine?

4. At what time was there the greatest change to the number of bottles in the machine? How did that change affect the number of remaining bottles in the machine?

5. At which time period, 8:05–8:09 or 8:25–8:29, was there a greater change to the number of bottles in the machine? Explain your reasoning.

6. The machine must be emptied to be serviced. If there are 40 bottles in the machine when it is to be serviced, what number will go in the second column in the table?

time	number of bottles
8:00–8:04	-1
8:05–8:09	+12
8:10–8:14	-4
8:15–8:19	-1
8:20–8:24	-5
8:25–8:29	-12
8:30–8:34	-2
8:35–8:39	0
8:40–8:40	0
8:45–8:49	-6
8:50–8:54	+24
8:55–8:59	0
service	

Are you ready for more?

Priya, Mai, and Lin went to a cafe on a weekend. Their shared bill came to $25. Each student gave the server a $10 bill. The server took this $30 and brought back five $1 bills in change. Each student took $1 back, leaving the rest, $2, as a tip for the server.

As she walked away from the cafe, Lin thought, "Wait—this doesn't make sense. Since I put in $10 and got $1 back, I wound up paying $9. So did Mai and Priya. Together, we paid $27. Then we left a $2 tip. That makes $29 total. And yet we originally gave the waiter $30. Where did the extra dollar go?"

Think about the situation and about Lin's question. Do you agree that the numbers didn't add up properly? Explain your reasoning.

Lesson 5 Summary

Sometimes we represent changes in a quantity with positive and negative numbers. If the quantity increases, the change is positive. If it decreases, the change is negative.

- Suppose 5 gallons of water is put in a washing machine. We can represent the change in the number of gallons as +5. If 3 gallons is emptied from the machine, we can represent the change as -3.

It is especially common to represent money we receive with positive numbers and money we spend with negative numbers.

- Suppose Clare gets $30.00 for her birthday and spends $18.00 buying lunch for herself and a friend. To her, the value of the gift can be represented as +30.00 and the value of the lunch as -18.00.

Whether a number is considered positive or negative depends on a person's perspective. If Clare's grandmother gives her $20 for her birthday, Clare might see this as +20, because to her, the amount of money she has increased. But her grandmother might see it as -20, because to her, the amount of money she has decreased.

In general, when using positive and negative numbers to represent changes, we have to be very clear about what it means when the change is positive and what it means when the change is negative.

NAME DATE PERIOD

Cool Down

Lesson 5: Using Negative Numbers to Make Sense of Contexts

Cool Down: Bakery Owner

The table shows records of money-related activities of a bakery owner over a period of a week.

date	items	amount in dollars
May 1	rent	-850.00
May 2	order (birthday cake and cookies)	106.75
May 3	utilities (electricity, gas, phone)	-294.50
May 5	order (wedding cake and desserts)	240.55
May 5	baking supplies	-147.95
May 6	order (anniversary cake)	158.20
May 7	order (breads and desserts for a conference)	482.30
May 7	bakery sales	415.65

1. For which items did she receive money?
2. What does the number -147.95 mean in this context?
3. Did the bakery owner receive more or spend more money on May 5? Explain how you know.

Unit 7 Lesson 5 Cumulative Practice Problems

1. Write a positive or negative number to represent each change in the high temperature.

   a. Tuesday's high temperature was 4 degrees less than Monday's high temperature.

   b. Wednesday's high temperature was 3.5 degrees less than Tuesday's high temperature.

   c. Thursday's high temperature was 6.5 degrees more than Wednesday's high temperature.

   d. Friday's high temperature was 2 degrees less than Thursday's high temperature.

2. Decide which of the following quantities can be represented by a positive number and which can be represented by a negative number. Give an example of a quantity with the opposite sign in the same situation.

   a. Tyler's puppy gained 5 pounds.

   b. The aquarium leaked 2 gallons of water.

   c. Andre received a gift of $10.

   d. Kiran gave a gift of $10.

   e. A climber descended 550 feet.

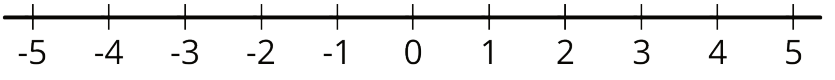
3. Make up a situation where a quantity is changing.

   a. Explain what it means to have a negative change.

   b. Explain what it means to have a positive change.

   c. Give an example of each.

4. a. On the number line, label the points that are 4 units away from 0.



   b. If you fold the number line so that a vertical crease goes through 0, the points you label would match up. Explain why this happens.

   c. On the number line, label the points that are $\frac{5}{2}$ units from 0. What is the distance between these points?

(From Unit 7, Lesson 2.)

5. Evaluate each expression.

   a. $2^3 \cdot 3$

   b. $\frac{4^2}{2}$

   c. 3^1

   a. $6^2 \div 4$

   b. $2^3 - 2$

   c. $10^2 + 5^2$

(From Unit 6, Lesson 12.)

Family Support Materials

Rational Numbers

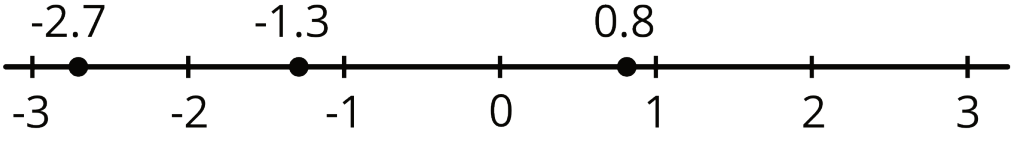
Negative Numbers and Absolute Value

Family Support Materials 1

This week, your student will work with signed numbers, or positive and negative numbers. We often compare signed numbers when talking about temperatures. For example, -30 degrees Fahrenheit is colder than -10 degrees Fahrenheit. We say "-30 is less than -10" and write: $-30 < -10$.

We also use signed numbers when referring to elevation, or height relative to the sea level. An elevation of 2 feet (which means 2 feet above sea level) is higher than an elevation of -4 feet (which means 4 feet below sea level). We say "2 is greater than -4" and write $2 > -4$.

We can plot positive and negative numbers on the number line. Numbers to the left are always less than numbers to the right.



We can see that -1.3 is less than 0.8 because -1.3 is to the left of 0.8, but -1.3 is greater than -2.7 because it is to the right of -2.7.

We can also talk about a number in terms of its **absolute value**, or its distance from zero on the number line. For example, 0.8 is 0.8 units away from zero, which we can write as $|0.8| = 0.8$, and -2.7 is 2.7 units away from zero, which we can write as $|-2.7| = 2.7$. The numbers -3 and 3 are both 3 units from 0, which we can write as $|3| = 3$ and $|-3| = 3$.

Here is a task to try with your student:

1. A diver is at the surface of the ocean, getting ready to make a dive. What is the diver's elevation in relation to sea level?

2. The diver descends 100 feet to the top of a wrecked ship. What is the diver's elevation now?

3. The diver descends 25 feet more toward the ocean floor. What is the absolute value of the diver's elevation now?

4. Plot each of the three elevations as a point on a number line. Label each point with its numeric value.

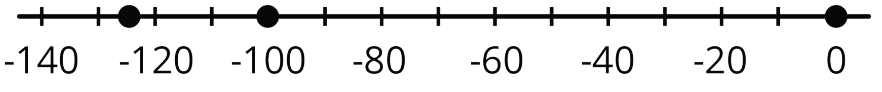
Solution:

1. 0, because sea level is 0 feet above or below sea level

2. -100, because the diver is 100 feet *below* sea level

3. The new elevation is -125 feet or 125 feet *below* sea level, so its absolute value is 125 feet.

4. A number line with 0, -100, and -125 marked, as shown:

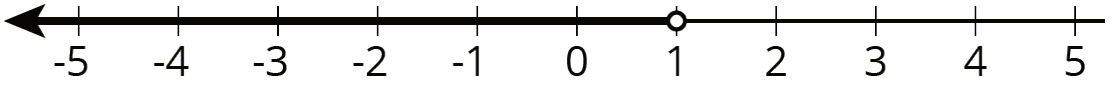


Inequalities

Family Support Materials 2

This week, your student will compare positive and negative numbers with inequalities symbols (< and >). They will also graph inequalities in one variable, such as $x < 1$ or $1 > x$, on the number line.

For example, to represent the statement "the temperature in Celsius (x) is less than 1 degree," we can write the inequality $x < 1$ and draw a number line like this:



The diagram shows all numbers to the left of 1 (or less than 1) being possible values of x.

We call any value of x that makes an inequality true a **solution to the inequality**.

This means x values that are greater than -8 are solutions to the inequality $x > -8$. Likewise, x values that are less than 15 could be a solution to the inequality $x < 15$. Depending on the context, however, the solutions may include only positive whole numbers (for example, if x represents the number of students in a class), or any positive and negative numbers, not limited to whole numbers (for example, if x represents temperatures).

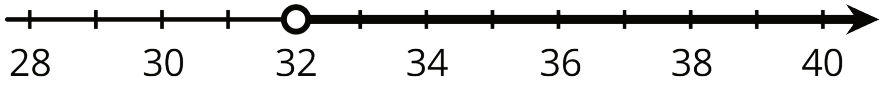
Here is a task to try with your student:

A sign at a fair says, "You must be taller than 32 inches to ride the ferris wheel." Write and graph an inequality that shows the heights of people who are tall enough to ride the ferris wheel.

Solution:

If x represents the height of a person in inches, then the inequality $x > 32$ represents the heights of people who can ride the ferris wheel. We can also write the inequality $32 < x$.

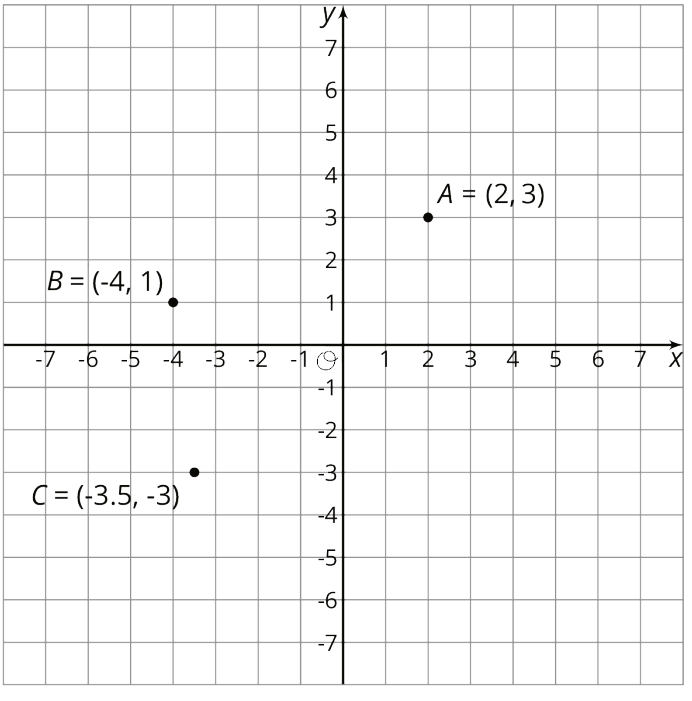
The graph of the inequality is:



The Coordinate Plane

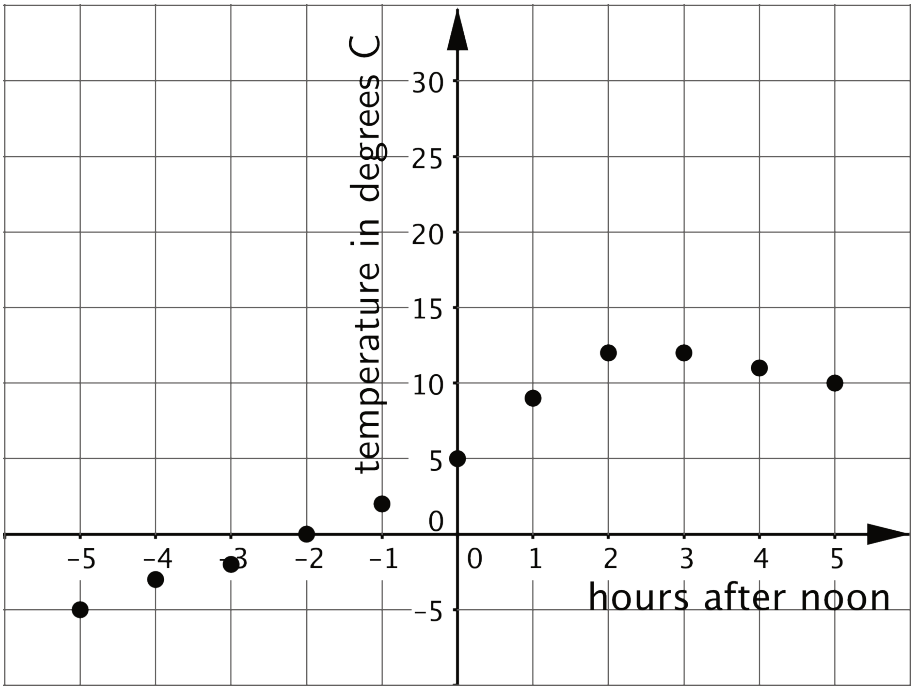
Family Support Materials 3

This week, your student will plot and interpret points on the coordinate plane. In earlier grades, they plotted points where both coordinates are positive, such as point A in the figure. They will now plot points that have positive and negative coordinates, such as points B and C.



To find the distance between two points that share the same horizontal line or the same vertical lines, we can simply count the grid units between them. For example, if we plot the point $(2, -4)$ on the grid above (try it!), we can tell that the point will be 7 units away from point $A = (2, 3)$.

Points on a coordinate plane can also represent situations that involve positive and negative numbers. For instance, the points on this coordinate plane shows the temperature in degrees Celsius every hour before and after noon on a winter day. Times before noon are negative and times after noon are positive.



For example, the point $(5, 10)$ tells us that 5 hours after noon, or 5:00 p.m, the temperature was 10 degrees Celsius.

Here is a task to try with your student:

In the graph of temperatures above:

1. What was the temperature at 7 a.m.?

2. For which recorded times was it colder than 5 degrees Celsius?

Solution:

1. It was -5 degrees Celsius at 7:00 a.m. You can see this at the point $(-5, -5)$.

2. It was 5 degrees Celsius right at noon, and for the times recorded before that, it was colder.

Common Factors and Common Multiples

Family Support Materials 4

This week, your student will solve problems that involve **factors** and **multiples**. Because $2 \cdot 6 = 12$, we say that 2 and 6 are factors of 12, and that 12 is a multiple of both 2 and 6. The number 12 has other factors: 1, 3, 4, and 12 itself.

Factors and multiples were studied in earlier grades. The focus here is on **common factors** and **common multiples** of two whole numbers. For example, 4 is a factor of 8 and a factor of 20, so 4 is a common factor of 8 and 20. 80 is a multiple of 8 and a multiple of 20, so 80 is a common multiple of those two numbers.

One way to find the common factors of two numbers is to list all of the factors for each number and see which factors they have in common. Sometimes we want to find the *greatest* common factor. To find the greatest common factor of 18 and 24, we first list all the factors of each number and look for the greatest one they have in common.

- Factors of 18: **1**, **2**, **3**, **6**, 9,18
- Factors of 24: **1**, **2**, **3**, 4, **6**, 8, 12, 24

The common factors are 1, 2, 3, and 6. Of these, 6 is the greatest one, so 6 is the greatest common factor of 18 and 24.

To find the common multiples of two numbers, we can do the same. Sometimes we want to find the *least* common multiple. Let's find the least common multiple of 18 and 24.

- Multiples of 18: 18, 36, 54, **72**, 90, 108, 126, **144**, . . .
- Multiples of 24: 24, 48, **72**, 96, 120, **144**, 168, 192, . . .

The first two common multiples are 72 and 144. We can see that 72 is the least common multiple.

Here is a task to try with your student:

A cook is making cheese sandwiches to sell. A loaf of bread can make 10 sandwiches. A package of cheese can make 15 sandwiches. How many loaves of bread and how many packages of cheese should the cook buy so that he can make cheese sandwiches without having any bread or any cheese left over?

Solution:

If he is using up the entire loaf of bread, then the number of sandwiches he can make will be a multiple of 10: 10, 20, **30**, 40, 50, **60**, 70, 80, **90**, 100, . . .

If he is using up all of the cheese in each package, then the number of sandwiches he can make will be a multiple of 15: 15, **30**, 45, **60**, 75, **90**, 105, . . .

30, 60, and 90 are some of the common multiples.

- To make 30 sandwiches, he will need 3 loaves of bread ($3 \cdot 10 = 30$) and 2 packages of cheese ($2 \cdot 15 = 30$).
- To make 60 sandwiches, he will need 6 loaves of bread and 4 packages of cheese.
- To make 90 sandwiches, he will need 9 loaves of bread and 6 packages of cheese.

There are other solutions as well! If he wants to buy the fewest number of loaves and cheese packages, then the first solution is the least.