Illustrative Mathematics

Grade 7

Unit 8

Probability and Sampling

Lessons 1-5

You can access the lessons on LearnZillion on Clever

NAME DATE PERIOD

Student Task Statements

Lesson 1: Mystery Bags

1.1: Going Fishing

Andre and his dad have been fishing for 2 hours. In that time, they have caught 9 bluegills and 1 yellow perch.

The next time Andre gets a bite, what kind of fish do you think it will be? Explain your reasoning.

1.2: Playing the Block Game

Your teacher will give your group a bag of colored blocks.

1. Follow these instructions to play one round of the game:
    a. Everyone in the group records the color written on the bag in the first column of the table.
    b. Without looking in the bag, one person takes out one of the blocks and shows it to the group.
    c. If they get a block that is the same color as the bag, they earn:
        - 1 point during round 1
        - 2 points during round 2
        - 3 points during round 3
    d. Next, they put the block back into the bag, shake the bag to mix up the blocks, and pass the bag to the next person in the group.
    e. Repeat these steps until everyone in your group has had 4 turns.
2. At the end of the round, record each person's score in the table.

	What color bag?	person 1's score	person 2's score	person 3's score	person 4's score
round 1					

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round 2

round 3

3. Pause here so your teacher can give you a new bag of blocks for the next round.
4. Repeat the previous steps to play rounds 2 and 3 of the game.
5. After you finish playing all 3 rounds, calculate the total score for each person in your group.

Are you ready for more?

Tyler's class played the block game using purple, orange, and yellow bags of blocks.

- During round 1, Tyler's group picked 4 purple blocks and 12 blocks of other colors.
- During round 2, Tyler's group picked 11 orange blocks and 5 blocks of other colors.
- During round 3, Tyler forgot to record how many yellow blocks his group picked.

For a final round, Tyler's group can pick one block from any of the three bags. Tyler's group decides that picking from the orange bag would give them the best chance of winning, and that picking from the purple bag would give them the worst chance of winning. What results from the yellow bag could have lead Tyler's group to this conclusion? Explain your reasoning.

Lesson 1 Summary

One of the main ways that humans learn is by repeating experiments and observing the results. Babies learn that dropping their cup makes it hit the floor with a loud noise by repeating this action over and over. Scientists learn about nature by observing the results of repeated experiments again and again. With enough data about the results of experiments, we can begin to predict what may happen if the experiment is repeated in the future. For example, a baseball player who has gotten a hit 33 out of 100 times at bat might be expected to get a hit about 33% of his times at bat in the future as well.

In some cases, we can predict the chances of things happening based on our knowledge of the situation. For example, a coin should land heads up about 50% of the time due to the symmetry of the coin.

In other cases, there are too many unknowns to predict the chances of things happening. For example, the chances of rain tomorrow are based on similar weather conditions we have observed in the past. In these situations, we can experiment, using past results to estimate chances.

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Lesson 1: Mystery Bags

Cool Down: Jada Draws Even

A large fish tank is filled with table tennis balls with numbers written on them. Jada chooses 10 table tennis balls from the tank and writes down their numbers.

1 3 5 1 3 2 4 1 5 3

A second tank is filled with golf balls with numbers written on them. Jada chooses 10 golf balls from the tank and writes down their numbers.

1 4 5 2 6 2 2 1 4 8

To win a prize, Jada must get a ball with an even number. Should she try to win the prize using the tank of table tennis balls or the tank of golf balls? Explain your reasoning.

Unit 8 Lesson 1 Cumulative Practice Problems

1. Lin is interested in how many of her classmates watch her favorite TV show, so she starts asking around at lunch. She gets the following responses:

   yes yes yes no no no no no

   no no yes no no no

   If she asks one more person randomly in the cafeteria, do you think they will say "yes" or "no"? Explain your reasoning.

2. An engineer tests the strength of a new material by seeing how much weight it can hold before breaking. Previous tests have held these weights in pounds:

   1,200 1,400 1,300 1,500 950 1,600 1,100

   Do you think that this material will be able to hold more than 1,000 pounds in the next test? Explain your reasoning.

3. A company tests two new products to make sure they last for more than a year.

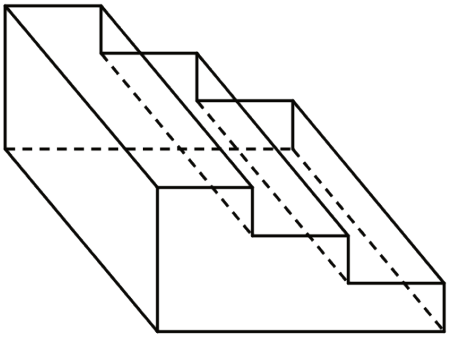
   - Product 1 had 950 out of 1,000 test items last for more than a year.
   - Product 2 had 150 out of 200 last for more than a year.

   If you had to choose one of these two products to use for more than a year, which one is more likely to last? Explain your reasoning.

4. Put these numbers in order from least to greatest.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{5}$ 0.6 0.3

5. A small staircase is made so that the horizontal piece of each step is 10 inches long and 25 inches wide. Each step is 5 inches above the previous one. What is the surface area of this staircase?



(From Unit 7, Lesson 15.)

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Student Task Statements

Lesson 2: Chance Experiments

2.1: Which is More Likely?

Which is more likely to happen?

- When reaching into a dark closet and pulling out one shoe from a pile of 20 pairs of shoes, you pull out a left shoe.
- When listening to a playlist—which has 5 songs on it—in shuffle mode, the first song on the playlist plays first.

2.2: How Likely Is It?

1. Label each event with one of these options:

   impossible, unlikely, equally likely as not, likely, certain

   a. You will win grand prize in a raffle if you purchased 2 out of the 100 tickets.

   b. You will wait less than 10 minutes before ordering at a fast food restaurant.

   c. You will get an even number when you roll a standard number cube.

   d. A four-year-old child is over 6 feet tall.

   e. No one in your class will be late to class next week.

   f. The next baby born at a hospital will be a boy.

   g. It will snow at our school on July 1.

   h. The Sun will set today before 11:00 p.m.

   i. Spinning this spinner will result in green.

   j. Spinning this spinner will result in red.

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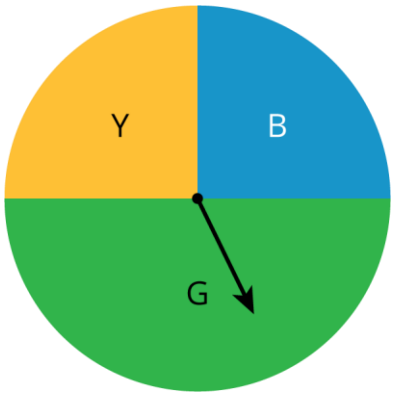

2. Discuss your answers to the previous question with your partner. If you disagree, work to reach an agreement.

3. Invent another situation for each label, for a total of 5 more events.

2.3: Take a Chance

Your teacher will have 2 students play a short game.

1. When the first person chose 3 numbers, did they usually win?

2. When the person chose 4 numbers, did you expect them to win? Explain your reasoning.

Are you ready for more?

On a game show, there are 3 closed doors. One door has a prize behind it. The contestant chooses one of the doors. The host of the game show, who knows where the prize is located, opens one of the *other* doors which does not have the prize. The contestant can choose to stay with their first choice or switch to the remaining closed door.

1. Do you think it matters if the contestant switches doors or stays?

2. Practice playing the game with your partner and record your results. Whoever is the host starts each round by secretly deciding which door has the prize.

    a. Play 20 rounds where the contestant always stays with their first choice.

    b. Play 20 more rounds where the contestant always switches doors.

3. Did the results from playing the game change your answer to the first question? Explain.

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2.4: Card Sort: Likelihood

1. Your teacher will give you some cards that describe events. Order the events from least likely to most likely.
2. After ordering the first set of cards, pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.
3. Add the new set of cards to the first set so that all of the cards are ordered from least likely to most likely.

Lesson 2 Summary

A **chance experiment** is something that happens where the outcome is unknown. For example, if we flip a coin, we don't know if the result will be a head or a tail. An **outcome** of a chance experiment is something that can happen when you do a chance experiment. For example, when you flip a coin, one possible outcome is that you will get a head. An **event** is a set of one or more outcomes.

We can describe events using these phrases:

- Impossible
- Unlikely
- Equally likely as not
- Likely
- Certain

For example, if you flip a coin:

- It is *impossible* that the coin will turn into a bottle of ketchup.
- It is *unlikely* the coin will land on its edge.
- It is *equally likely as not* that you will get a tail.
- It is *likely* that you will get a head or a tail.
- It is *certain* that the coin will land somewhere.

The *probability* of an event is a measure of the likelihood that an event will occur. We will learn more about probabilities in the lessons to come.

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Set 1 Likelihood

The weather report says there is a 20% chance of rain tomorrow. The chance of rain tomorrow.

Set 1 Likelihood

10% of people are left handed. The chance that a randomly chosen person is left handed.

Set 1 Likelihood

The offspring of two fruit flies in a science experiment have a 75% chance of having red eyes. The chance that the first fly to hatch has red eyes.

Set 1 Likelihood

Half of the cards in a deck are red and half are black. Shuffle the cards and select the first card. The chance that the card is red.

Set 2 Likelihood

2 out of every 5 dentists recommend a certain brand of toothpaste. The chance that a random dentist recommends the toothpaste.

Set 2 Likelihood

The chance that your opponent will play rock first in a game of paper, rock, scissors.

Set 2 Likelihood

A pile contains 6 square pattern blocks and you choose one. The chance that the block you choose has 4 sides of the same length.

Set 2 Likelihood

A fishbowl contains 5 balls where each one has an even number from 2 to 10 written on it and you choose one. The chance that you draw out a ball with the number 3 on it.

Set 2 Likelihood

In general English usage, $\frac{4}{25}$ of words begin with the letter T. The chance that a randomly chosen word in a novel begins with the letter T.

Set 2 Likelihood

The probability that a certain medical test gives the right result is 0.95. The chance that this medical test is correct for a random patient.

Lesson 2: Chance Experiments

Cool Down: According To

Here are some scenarios:

- According to market research, a business has a 75% chance of making money in the first 3 years.
- According to lab testing, $\frac{5}{6}$ of a certain kind of experimental light bulb will work after 3 years.
- According to experts, the likelihood of a car needing major repairs in the first 3 years is 0.7.

1. Write the scenarios in order of likelihood from least to greatest after three years: the business makes money, the light bulb still works, and the car needs major repairs.

2. Name another chance experiment that has the same likelihood as one of the scenarios.

Unit 8 Lesson 2 Cumulative Practice Problems

1. The likelihood that Han makes a free throw in basketball is 60%. The likelihood that he makes a 3-point shot is 0.345. Which event is more likely, Han making a free throw or making a 3-point shot? Explain your reasoning.

2. Different events have the following likelihoods. Sort them from least to greatest:

   60% 8 out of 10 0.37 20% $\frac{5}{6}$

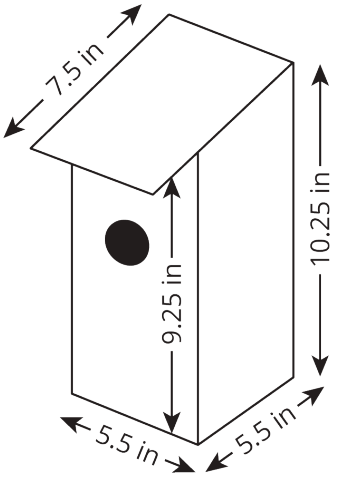
3. There are 25 prime numbers between 1 and 100. There are 46 prime numbers between 1 and 200. Which situation is more likely? Explain your reasoning.

   - A computer produces a random number between 1 and 100 that is prime.
   - A computer produces a random number between 1 and 200 that is prime.

4. It takes $4\frac{3}{8}$ cups of cheese, $\frac{7}{8}$ cups of olives, and $2\frac{5}{8}$ cups of sausage to make a signature pizza. How much of each ingredient is needed to make 10 pizzas? Explain or show your reasoning.

   (From Unit 4, Lesson 2.)

5. Here is a diagram of a birdhouse Elena is planning to build. (It is a simplified diagram, since in reality, the sides will have a thickness.) About how many square inches of wood does she need to build this birdhouse?



(From Unit 7, Lesson 16.)

6. Select **all** the situations where knowing the surface area of an object would be more useful than knowing its volume.

   A. Placing an order for tiles to replace the roof of a house.

   B. Estimating how long it will take to clean the windows of a greenhouse.

   C. Deciding whether leftover soup will fit in a container.

   D. Estimating how long it will take to fill a swimming pool with a garden hose.

   E. Calculating how much paper is needed to manufacture candy bar wrappers.

   F. Buying fabric to sew a couch cover.

   G. Deciding whether one muffin pan is enough to bake a muffin recipe.

(From Unit 7, Lesson 15.)

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Student Task Statements

Lesson 3: What Are Probabilities?

3.1: Which Game Would You Choose?

Which game would you choose to play? Explain your reasoning.

Game 1: You flip a coin and win if it lands showing heads.

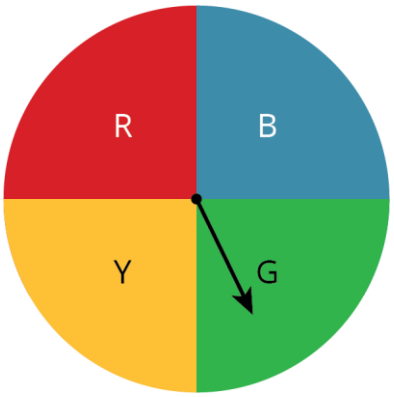
Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

3.2: What's Possible?

1. For each situation, list the **sample space** and tell how many outcomes there are.

    a. Han rolls a standard number cube once.

    b. Clare spins this spinner once.



    c. Kiran selects a letter at **random** from the word "MATH."

    d. Mai selects a letter at random from the alphabet.

    e. Noah picks a card at random from a stack that has cards numbered 5 through 20.

2. Next, compare the likelihood of these outcomes. Be prepared to explain your reasoning.

    a. Is Clare more likely to have the spinner stop on the red or blue section?

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b. Is Kiran or Mai more likely to get the letter T?

c. Is Han or Noah more likely to get a number that is greater than 5?

3. Suppose you have a spinner that is evenly divided showing all the days of the week. You also have a bag of papers that list the months of the year. Are you more likely to spin the current day of the week or pull out the paper with the current month?

Are you ready for more?

Are there any outcomes for two people in this activity that have the same likelihood? Explain or show your reasoning.

3.3: What's in the Bag?

Your teacher will give your group a bag of paper slips with something printed on them. Repeat these steps until everyone in your group has had a turn.

- As a group, guess what is printed on the papers in the bag and record your guess in the table.
- Without looking in the bag, one person takes out one of the papers and shows it to the group.
- Everyone in the group records what is printed on the paper.
- The person who took out the paper puts it back into the bag, shakes the bag to mix up the papers, and passes the bag to the next person in the group.

	Guess the sample space.	What is printed on the paper?
person 1		
person 2		
person 3		
person 4		

1. How was guessing the sample space the fourth time different from the first?
2. What could you do to get a better guess of the sample space?
3. Look at all the papers in the bag. Were any of your guesses correct?
4. Are all of the possible outcomes equally likely? Explain.
5. Use the sample space to determine the **probability** that a fifth person would get the same outcome as person 1.

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Lesson 3 Summary

The **probability** of an event is a measure of the likelihood that the event will occur. Probabilities are expressed using numbers from 0 to 1.

- If the probability is 0, that means the event is impossible. For example, when you flip a coin, the probability that it will turn into a bottle of ketchup is 0. The closer the probability of some event is to 0, the less likely it is.
- If the probability is 1, that means the event is certain. For example, when you flip a coin, the probability that it will land somewhere is 1. The closer the probability of some event is to 1, the more likely it is.

If we list all of the possible outcomes for a chance experiment, we get the **sample space** for that experiment. For example, the sample space for rolling a standard number cube includes six outcomes: 1, 2, 3, 4, 5, and 6. The probability that the number cube will land showing the number 4 is $\frac{1}{6}$. In general, if all outcomes in an experiment are equally likely and there are n possible outcomes, then the probability of a single outcome is $\frac{1}{n}$.

Sometimes we have a set of possible outcomes and we want one of them to be selected at **random**. That means that we want to select an outcome in a way that each of the outcomes is *equally likely*. For example, if two people both want to read the same book, we could flip a coin to see who gets to read the book first.

A	B	C
D	E	F
G	H	I
J	K	L
M	N	O

A	B	C
D	E	F
G	H	I
J	K	L
M	N	O

Lesson 3: What Are Probabilities?

Cool Down: Letter of the Day

A mother decides to teach her son about a letter each day of the week. She will choose a letter from the name of the day. For example, on Saturday she might teach about the letter S or the letter U, but not the letter M.

1. What letters are possible to teach using this method? (There are 15.)

2. What are 4 letters that can't be taught using this method?

3. On TUESDAY, the mother writes the word on a piece of paper and cuts it up so that each letter is on a separate piece of paper. She mixes up the papers and picks one. What is the probability that she will choose the piece of paper with the letter Y? Explain your reasoning.

Unit 8 Lesson 3 Cumulative Practice Problems

1. List the *sample space* for each chance experiment.

    a. Flipping a coin

    b. Selecting a random season of the year

    c. Selecting a random day of the week

2. A computer randomly selects a letter from the alphabet.

    a. How many different outcomes are in the sample space?

    b. What is the probability the computer produces the first letter of your first name?

3. What is the probability of selecting a random month of the year and getting a month that starts with the letter "J?" If you get stuck, consider listing the sample space.

4. E represents an object's weight on Earth and M represents that same object's weight on the Moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

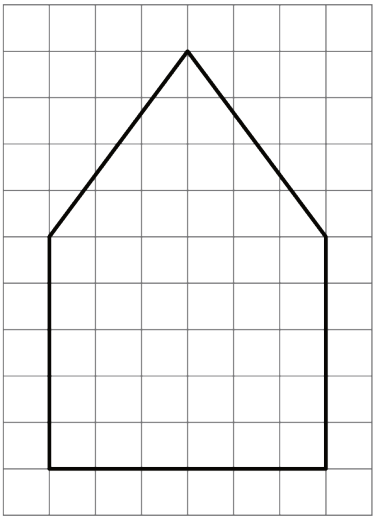
   a. What does the $\frac{1}{6}$ represent in this situation?

   b. Give an example of what a person might weigh on Earth and on the Moon.

(From Unit 2, Lesson 4.)

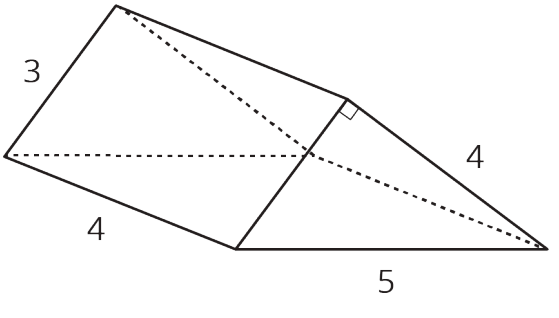
5. Here is a diagram of the base of a bird feeder which is in the shape of a pentagonal prism. Each small square on the grid is 1 square inch.

   The distance between the two bases is 8 inches. What will be the volume of the completed bird feeder?



(From Unit 7, Lesson 13.)

6. Find the surface area of the triangular prism.



(From Unit 7, Lesson 14.)

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Student Task Statements

Lesson 4: Estimating Probabilities Through Repeated Experiments

4.1: Decimals on the Number Line

1. Locate and label these numbers on the number line.

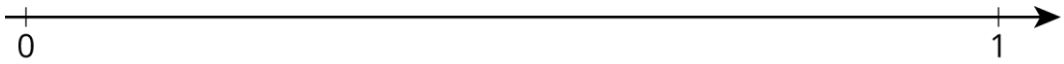
   a. 0.5

   b. 0.75

   c. 0.33

   d. 0.67

   e. 0.25



2. Choose one of the numbers from the previous question. Describe a game in which that number represents your probability of winning.

4.2: In the Long Run

Mai plays a game in which she only wins if she rolls a 1 or a 2 with a standard number cube.

1. List the outcomes in the sample space for rolling the number cube.
2. What is the probability Mai will win the game? Explain your reasoning.
3. If Mai is given the option to flip a coin and win if it comes up heads, is that a better option for her to win?
4. With your group, follow these instructions 10 times to create the graph.
   - One person rolls the number cube. Everyone records the outcome.
   - Calculate the fraction of rolls that are a win for Mai so far. Approximate the fraction with a decimal value rounded to the hundredths place. Record both the fraction and the decimal in the last column of the table.
   - On the graph, plot the number of rolls and the fraction that were wins.

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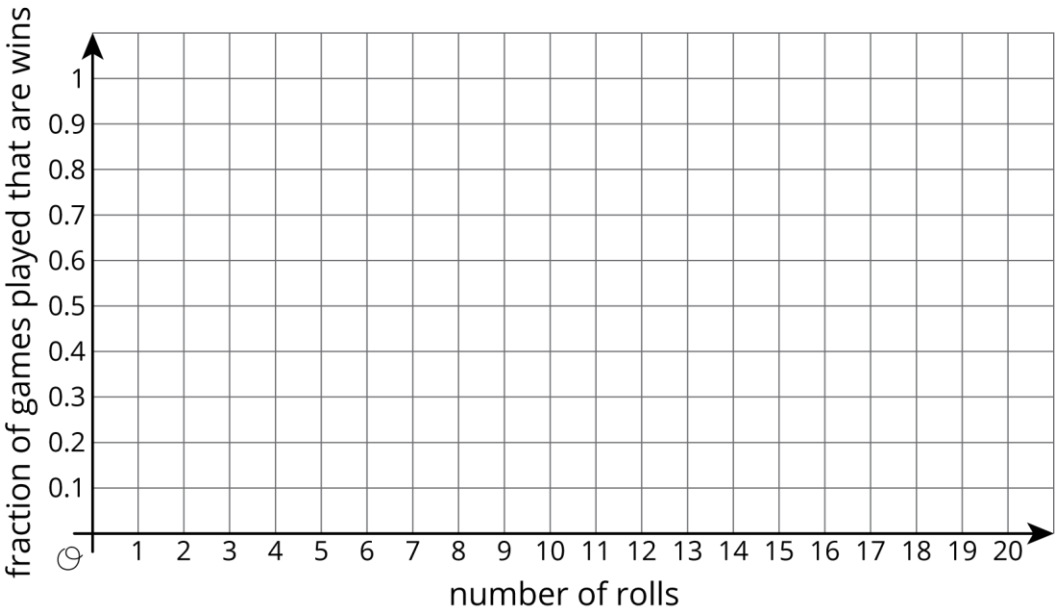
– Pass the number cube to the next person in the group.

roll	outcome	total number of wins for Mai	fraction of games played that are wins
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			



5. What appears to be happening with the points on the graph?

   a. After 10 rolls, what fraction of the total rolls were a win?

   b. How close is this fraction to the probability that Mai will win?

6. Roll the number cube 10 more times. Records your results in this table and on the graph from earlier.

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roll	outcome	total number of wins for Mai	fraction of games played that are wins
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

a. After 20 rolls, what fraction of the total rolls were a win?

b. How close is this fraction to the probability that Mai will win?

4.3: Due For a Win

1. For each situation, do you think the result is surprising or not? Is it possible? Be prepared to explain your reasoning.

   a. You flip the coin once, and it lands heads up.

   b. You flip the coin twice, and it lands heads up both times.

   c. You flip the coin 100 times, and it lands heads up all 100 times.

2. If you flip the coin 100 times, how many times would you expect the coin to land heads up? Explain your reasoning.

3. If you flip the coin 100 times, what are some other results that would not be surprising?

4. You've flipped the coin 3 times, and it has come up heads once. The cumulative fraction of heads is currently $\frac{1}{3}$. If you flip the coin one more time, will it land heads up to make the cumulative fraction $\frac{2}{4}$?

Lesson 4 Summary

A probabilityfor an event represents the proportion of the time we expect that event to occur in the long run. For example, the probability of a coin landing heads up after a flip is $\frac{1}{2}$,

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which means that if we flip a coin many times, we expect that it will land heads up about half of the time.

Even though the probability tells us what we should expect if we flip a coin many times, that doesn't mean we are more likely to get heads if we just got three tails in a row. The chances of getting heads are the same every time we flip the coin, no matter what the outcome was for past flips.

Lesson 4: Estimating Probabilities Through Repeated Experiments

Cool Down: Fiction or Non-fiction?

A librarian is curious about the habits of the library's patrons. He records the type of item that the first 10 patrons check out from the library.

patron	item type
1	fiction book
2	non-fiction book
3	fiction book
4	fiction book
5	audiobook
6	non-fiction book
7	DVD
8	non-fiction book
9	fiction book
10	DVD

Based on the information from these patrons . . .

1. Estimate the probability that the next patron will check out a fiction book. Explain your reasoning.

2. Estimate the number of DVDs that will be checked out for every 100 patrons. Explain your reasoning.

Unit 8 Lesson 4 Cumulative Practice Problems

1. A carnival game has 160 rubber ducks floating in a pool. The person playing the game takes out one duck and looks at it.

    - If there's a red mark on the bottom of the duck, the person wins a small prize.
    - If there's a blue mark on the bottom of the duck, the person wins a large prize.
    - Many ducks do not have a mark.

    After 50 people have played the game, only 3 of them have won a small prize, and none of them have won a large prize.

    Estimate the number of the 160 ducks that you think have red marks on the bottom. Then estimate the number of ducks you think have blue marks. Explain your reasoning.

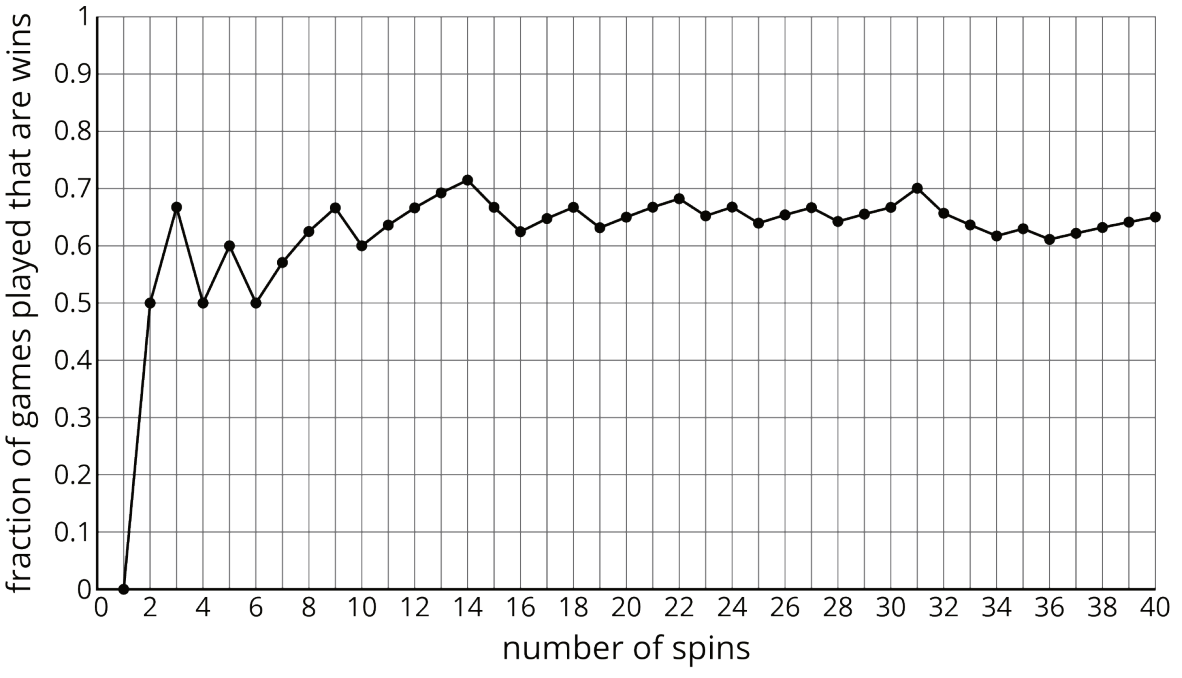
2. Lin wants to know if flipping a quarter really does have a probability of $\frac{1}{2}$ of landing heads up, so she flips a quarter 10 times. It lands heads up 3 times and tails up 7 times. Has she proven that the probability is not $\frac{1}{2}$? Explain your reasoning.

3. A spinner has four equal sections, with one letter from the word "MATH" in each section.

    a. You spin the spinner 20 times. About how many times do you expect it will land on A?

    b. You spin the spinner 80 times. About how many times do you expect it will land on something other than A? Explain your reasoning.

4. A spinner is spun 40 times for a game. Here is a graph showing the fraction of games that are wins under some conditions.



Estimate the probability of a spin winning this game based on the graph.

5. Which event is more likely: rolling a standard number cube and getting an even number, or flipping a coin and having it land heads up?

(From Unit 8, Lesson 2.)

6. Noah will select a letter at random from the word "FLUTE." Lin will select a letter at random from the word "CLARINET."

Which person is more likely to pick the letter "E?" Explain your reasoning.

(From Unit 8, Lesson 3.)

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Student Task Statements

Lesson 5: More Estimating Probabilities

5.1: Is it Likely?

1. If the weather forecast calls for a 20% chance of light rain tomorrow, would you say that it is likely to rain tomorrow?
2. If the probability of a tornado today is $\frac{1}{10}$, would you say that there will likely be a tornado today?
3. If the probability of snow this week is 0.85, would you say that it is likely to snow this week?

5.2: Making My Head Spin

Your teacher will give you 4 spinners. Make sure each person in your group uses a different spinner.

1. Spin your spinner 10 times, and record your outcomes.
2. Did you get all of the different possible outcomes in your 10 spins?
3. What fraction of your 10 spins landed on 3?
4. Next, share your outcomes with your group, and record their outcomes.
    a. Outcomes for spinner A:
    b. Outcomes for spinner B:
    c. Outcomes for spinner C:
    d. Outcomes for spinner D:
5. Do any of the spinners have the same sample space? If so, do they have the same probabilities for each number to result?
6. For each spinner, what is the probability that it lands on the number 3? Explain or show your reasoning.
7. For each spinner, what is the probability that it lands on something other than the number 3? Explain or show your reasoning.

NAME DATE PERIOD

8. Noah put spinner D on top of his closed binder and spun it 10 times. It never landed on the number 1. How might you explain why this happened?

9. Han put spinner C on the floor and spun it 10 times. It never landed on the number 3, so he says that the probability of getting a 3 is 0. How might you explain why this happened?

Are you ready for more?

Design a spinner that has a $\frac{2}{3}$ probability of landing on the number 3. Explain how you could precisely draw this spinner.

5.3: How Much Green?

Your teacher will give you a bag of blocks that are different colors. Do not look into the bag or take out more than 1 block at a time. Repeat these steps until everyone in your group has had 4 turns.

- Take one block out of the bag and record whether or not it is green.
- Put the block back into the bag, and shake the bag to mix up the blocks.
- Pass the bag to the next person in the group.

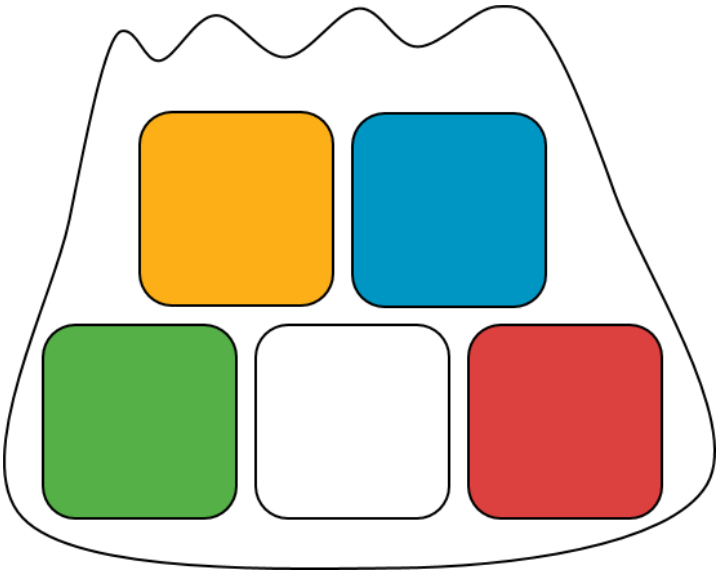
1. What do you think is the probability of taking out a green block from this bag? Explain or show your reasoning.

2. How could you get a better estimate without opening the bag?

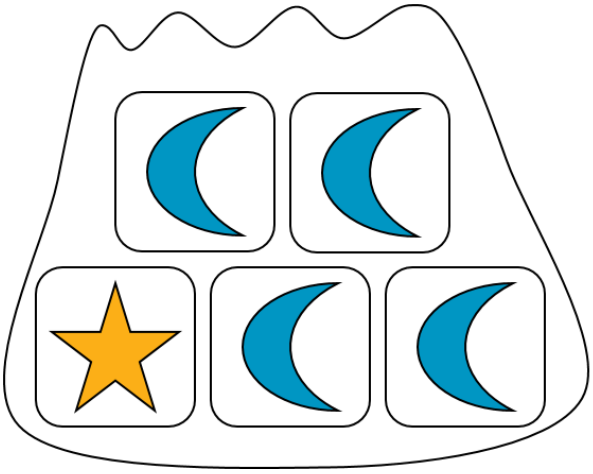
Lesson 5 Summary

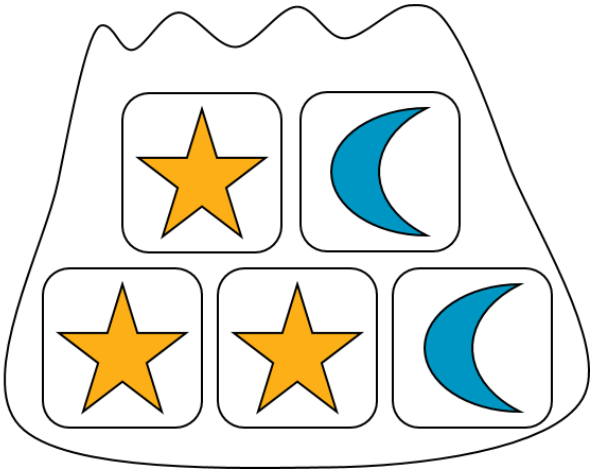
Suppose a bag contains 5 blocks. If we select a block at random from the bag, then the probability of getting any one of the blocks is $\frac{1}{5}$.

NAME DATE PERIOD



Now suppose a bag contains 5 blocks. Some of the blocks have a star, and some have a moon. If we select a block from the bag, then we will either get a star block or a moon block. The probability of getting a star block depends on how many there are in the bag.



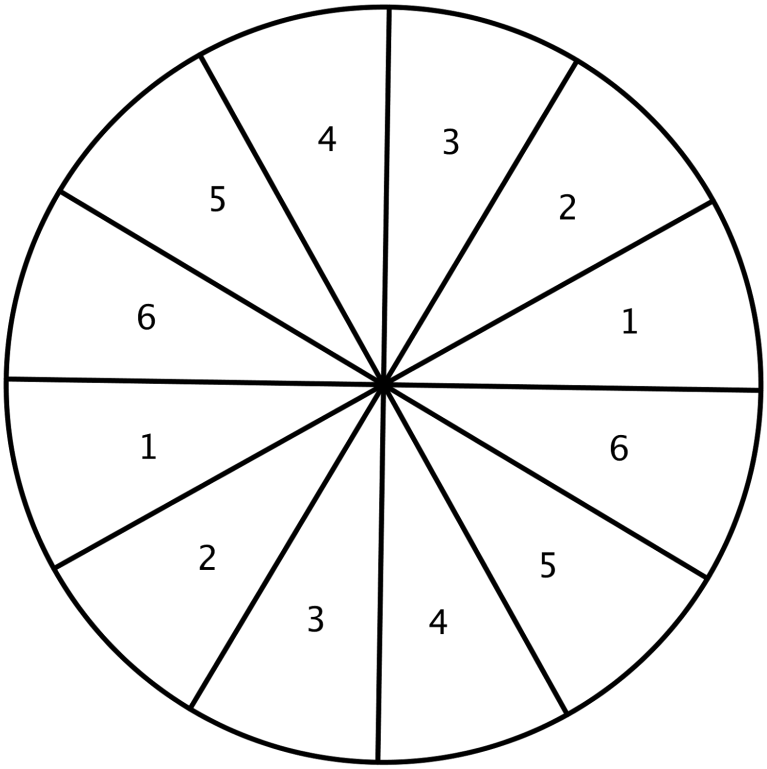


In this example, the probability of selecting a star block at random from the first bag is $\frac{1}{5}$, because it contains only 1 star block. (The probability of getting a moon block is $\frac{4}{5}$.) The probability of selecting a star block at random from the second bag is $\frac{3}{5}$, because it contains 3 star blocks. (The probability of getting a moon block from this bag is $\frac{2}{5}$.)

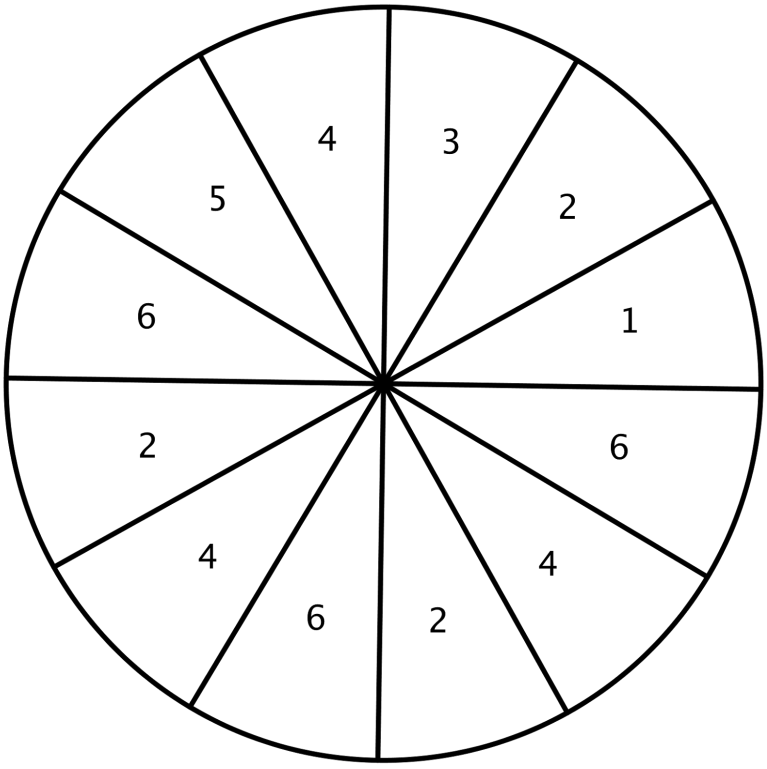
This shows that two experiments can have the same sample space, but different probabilities for each outcome.

NAME DATE PERIOD

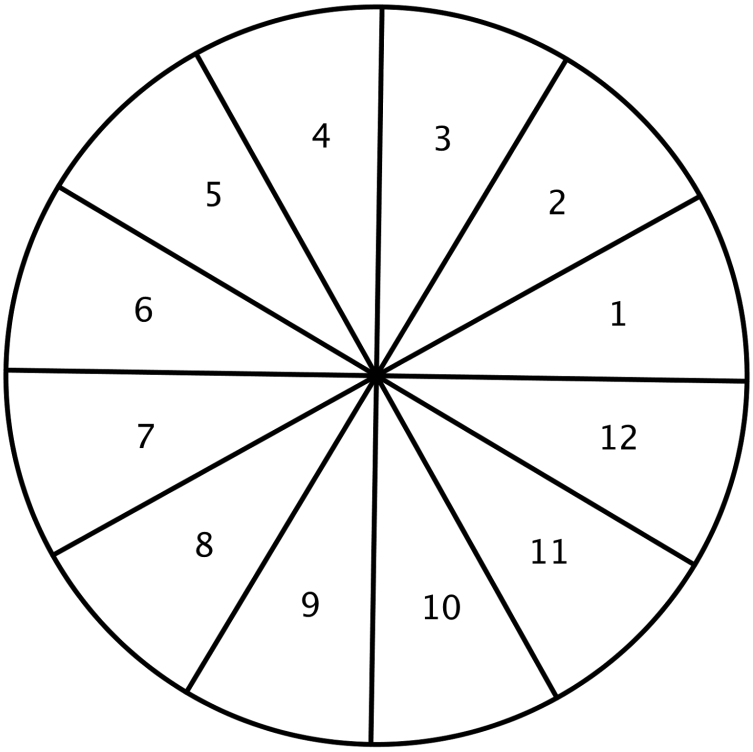
Spinner A



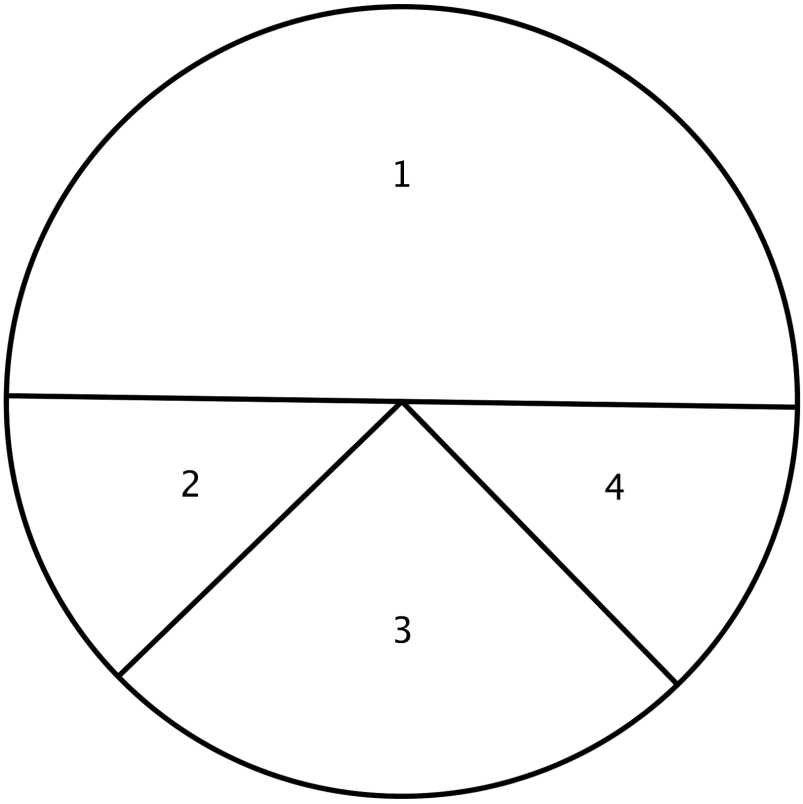
Spinner B



Spinner C



Spinner D



Lesson 5: More Estimating Probabilities

Cool Down: The Probability of Spinning B

Jada, Diego, and Elena each use the same spinner that has four (not necessarily equal sized) sections marked A, B, C, and D.

- Jada says, "The probability of spinning B is 0.3 because I spun 10 times, and it landed on B 3 times."
- Diego says, "The probability of spinning B is 20% because I spun 5 times, and it landed on B once."
- Elena says, "The probability of spinning B is $\frac{2}{7}$ because I spun 7 times, and it landed on B twice."

1. Based on their methods, which probability estimate do you think is the most accurate? Explain your reasoning.

2. Andre measures the spinner and finds that the B section takes up $\frac{1}{4}$ of the circle. Explain why none of the methods match this probability exactly.

Unit 8 Lesson 5 Cumulative Practice Problems

1. What is the same about these two experiments? What is different?

    - Selecting a letter at random from the word “ALABAMA”
    - Selecting a letter at random from the word “LAMB”

2. Andre picks a block out of a bag 60 times and notes that 43 of them were green.

    a. What should Andre estimate for the probability of picking out a green block from this bag?

    b. Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your reasoning.

3. Han has a number cube that he suspects is not so standard.

    - Han rolls the cube 100 times, and it lands on a six 40 times.
    - Kiran rolls the cube 50 times, and it lands on a six 21 times.
    - Lin rolls the cube 30 times, and it lands on a six 11 times.

    Based on these results, is there evidence to help prove that this cube is not a standard number cube? Explain your reasoning.

4. A textbook has 428 pages numbered in order starting with 1. You flip to a random page in the book in a way that it is equally likely to stop at any of the pages.

   a. What is the sample space for this experiment?

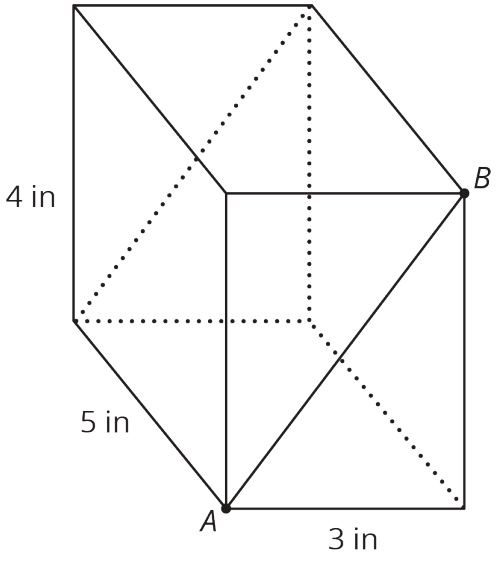
   b. What is the probability that you turn to page 45?

   c. What is the probability that you turn to an even numbered page?

   d. If you repeat this experiment 50 times, about how many times do you expect you will turn to an even numbered page?

(From Unit 8, Lesson 3.)

5. A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between A and B is 5 inches.



What is the surface area of the original rectangular prism? What is the total surface area of the two triangular prisms together?

(From Unit 7, Lesson 15.)

Family Support Materials

Probability and Sampling

Probabilities of Single Step Events

Family Support Materials 1

This week your student will be working with probability. A **probability** is a number that represents how likely something is to happen. For example, think about flipping a coin.

- The probability that the coin lands somewhere is 1. That is certain.
- The probability that the coin lands heads up is $\frac{1}{2}$, or 0.5.
- The probability that the coin turns into a bottle of ketchup is 0. That is impossible.

Sometimes we can figure out an exact probability. For example, if we pick a random date, the chance that it is on a weekend is $\frac{2}{7}$, because 2 out of every 7 days fall on the weekend. Other times, we can estimate a probability based on what we have observed in the past.

Here is a task to try with your student:

People at a fishing contest are writing down the type of each fish they catch. Here are their results:

- Person 1: bass, catfish, catfish, bass, bass, bass
- Person 2: catfish, catfish, bass, bass, bass, bass, catfish, catfish, bass, catfish
- Person 3: bass, bass, bass, catfish, bass, bass, catfish, bass, catfish

1. Estimate the probability that the next fish that gets caught will be a bass.
2. Another person in the competition caught 5 fish. Predict how many of these fish were bass.
3. Before the competition, the lake was stocked with equal numbers of catfish and bass. Describe some possible reasons for why the results do not show a probability of $\frac{1}{2}$ for catching a bass.

Solution:

1. About $\frac{15}{25}$, or 0.6, because of the 25 fish that have been caught, 15 of them were bass.

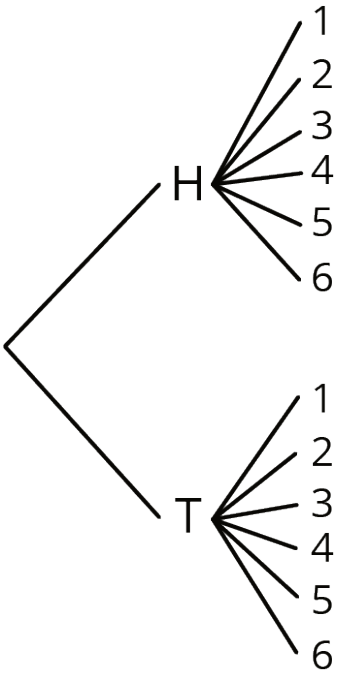
2. About 3 bass, because $\frac{3}{5} = 0.6$. It would also be reasonable if they caught 2 or 4 bass, out of their 5 fish.

3. There are many possible answers. For example:
   - Maybe the lures or bait they were using are more likely to catch bass.
   - With results from only 25 total fish caught, we can expect the results to vary a little from the exact probability.

Probabilities of Multi-step Events

Family Support Materials 2

To find an exact probability, it is important to know what outcomes are possible. For example, to show all the possible outcomes for flipping a coin and rolling a number cube, we can draw this tree diagram:



The branches on this tree diagram represent the 12 possible outcomes, from "heads 1" to "tails 6." To find the probability of getting heads on the coin and an even number on the number cube, we can see that there are 3 ways this could happen ("heads 2", "heads 4", or "heads 6") out of 12 possible outcomes. That means the probability is $\frac{3}{12}$, or 0.25.

Here is a task to try with your student:

A board game uses cards that say "forward" or "backward" and a spinner numbered from 1 to 5.

1. On their turn, a person picks a card and spins the spinner to find out which way and how far to move their piece. How many different outcomes are possible?

2. On their next turn, what is the probability that the person will:
   a. get to move their piece forward 5 spaces?

   b. have to move their piece backward some odd number of spaces?

Solution:

1. There are 10 possible outcomes (“forward 1”, “forward 2”, “forward 3”, “forward 4”, “forward 5”, “backward 1”, “backward 2”, “backward 3”, “backward 4”, or “backward 5”).

2. a. $\frac{1}{10}$ or 0.1, because “forward 5” is 1 out of the 10 possibilities.

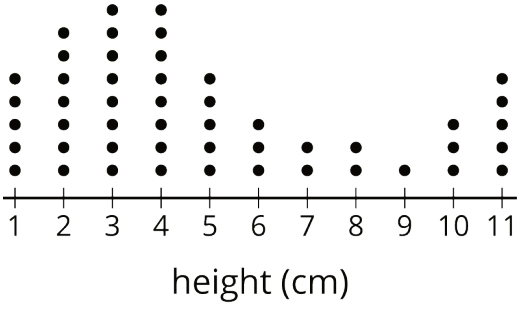
   b. $\frac{3}{10}$ or 0.3, because there are 3 such possibilities (“backward 1”, “backward 3”, or “backward 5”)

Sampling

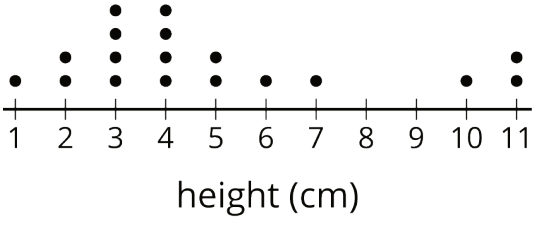
Family Support Materials 3

This week your student will be working with data. Sometimes we want to know information about a group, but the group is too large for us to be able to ask everyone. It can be useful to collect data from a **sample** (some of the group) of the **population** (the whole group). It is important for the sample to resemble the population.

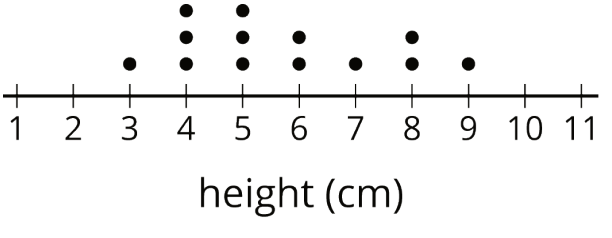
- For example, here is a dot plot showing a population: the height of 49 plants in a sprout garden.



- This sample is **representative** of the population, because it includes only a part of the data, but it still resembles the population in shape, center, and spread.



- This sample is not representative of the population. It has too many plant heights in the middle and not enough really short or really tall ones.



A sample that is selected at random is more likely to be representative of the population than a sample that was selected some other way.

Here is a task to try with your student:

A city council needs to know how many buildings in the city have lead paint, but they don't have enough time to test all 100,000 buildings in the city. They want to test a sample of buildings that will be representative of the population.

1. What would be a *bad* way to pick a sample of the buildings?

2. What would be a *good* way to pick a sample of the buildings?

Solution:

1. There are many possible answers.
    - Testing all the same type of buildings (like all the schools, or all the gas stations) would not lead to a representative sample of all the buildings in the city.
    - Testing buildings all in the same location, such as the buildings closest to city hall, would also be a bad way to get a sample.
    - Testing all the newest buildings would *bias* the sample towards buildings that don't have any lead paint.
    - Testing a small number of buildings, like 5 or 10, would also make it harder to use the sample to make predictions about the entire population.
2. To select a sample at random, they could put the addresses of all 100,000 buildings into a computer and have the computer select 50 addresses randomly from the list. Another possibility could be picking papers out a bag, but with so many buildings in the city, this method would be difficult.

Using Samples

Family Support Materials 4

We can use statistics from a sample (a part of the entire group) to estimate information about a population (the entire group). If the sample has more variability (is very spread out), we may not trust the estimate as much as we would if the numbers were closer together. For example, it would be easier to estimate the average height of all 3-year olds than all 40-year olds, because there is a wider range of adult heights.

We can also use samples to help predict whether there is a meaningful difference between two populations, or whether there is a lot of overlap in the data.

Here is a task to try with your student:

Students from seventh grade and ninth grade were selected at random to answer the question, "How many pencils do you have with you right now?" Here are the results:

how many pencils each seventh grade student had

4	1	2	5	2	1	1	2	3	3

how many pencils each ninth grade student had

9	4	1	14	6	2	0	8	2	5

1. Use the sample data to estimate the mean (average) number of pencils carried by:
   a. all the seventh grade students in the whole school.

   b. all the ninth grade students in the whole school.

2. Which sample had more variability? What does this tell you about your estimates in the previous question?

3. A student, who was not in the survey, has 5 pencils with them. If this is all you know, can you predict which grade they are in?

Solution:

1. Since the samples were selected at random, we predict they will represent the whole population fairly well.
   a. About 2.4 pencils for all seventh graders, because the mean of the sample is $(4 + 1 + 2 + 5 + 2 + 1 + 1 + 2 + 3 + 3) \div 10$ or 2.4 pencils.

b. About 5.1 pencils for all ninth graders, because the mean of the sample is $(9 + 4 + 1 + 14 + 6 + 2 + 0 + 8 + 2 + 5) \div 10$ or 5.1 pencils.

2. The survey of ninth graders had more variability. Those numbers were more spread out, so I trust my estimate for seventh grade more than I trust my estimate for ninth grade.

3. There are many possible answers. For example:
   - Since they only asked 10 students from each grade, it is hard to predict. It would help if they could ask more students.
   - The student is probably in ninth grade, because 5 is closer to the sample mean from ninth grade than from seventh grade.
   - The student could possibly be in seventh grade, because at least one student in seventh grade has 5 pencils.