Illustrative Mathematics

Grade 8
Exponents and Scientific Notation

Lessons 1-5

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for Syracuse City School District

Illustrative Mathematics

Lesson 1: Exponent Review

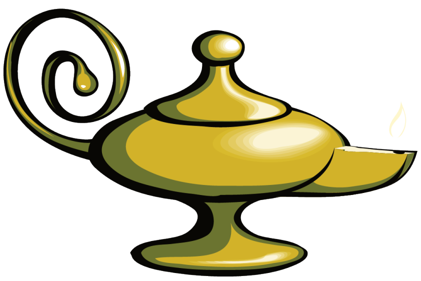
1.1: Which One Doesn’t Belong: Twos

Which expression does not belong? Be prepared to share your reasoning.

2^3	3^2
8	$2^2 \cdot 2^1$

1.2: Return of the Genie

Mai and Andre found an old, brass bottle that contained a magical genie. They freed the genie, and it offered them each a magical $1 coin as thanks.



- The magic coin turned into 2 coins on the first day.
- The 2 coins turned into 4 coins on the second day.
- The 4 coins turned into 8 coins on the third day.

This doubling pattern continued for 28 days.

Mai was trying to calculate how many coins she would have and remembered that instead of writing $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ for the number of coins on the 6th day, she could just write 2^6.

1. The number of coins Mai had on the 28th day is very, very large. Write an expression to represent this number without computing its value.

2. Andre’s coins lost their magic on the 25th day, so Mai has a lot more coins than he does. How many times more coins does Mai have than Andre?

1.3: Broken Coin

After a while, Jada picks up a coin that seems different than the others. She notices that the next day, only half of the coin is left!

- On the second day, only $\frac{1}{4}$ of the coin is left.
- On the third day, $\frac{1}{8}$ of the coin remains.

1. What fraction of the coin remains after 6 days?

2. What fraction of the coin remains after 28 days? Write an expression to describe this without computing its value.

3. Does the coin disappear completely? If so, after how many days?

Are you ready for more?

Every animal has two parents. Each of its parents also has two parents.

1. Draw a family tree showing an animal, its parents, its grandparents, and its great-grandparents.

2. We say that the animal's eight great-grandparents are "three generations back" from the animal. At which generation back would an animal have 262,144 ancestors?

Lesson 1 Summary

Exponents make it easy to show repeated multiplication. For example,

$$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2.$$

One advantage to writing 2^6 is that we can see right away that this is 2 to the *sixth* power. When this is written out using multiplication, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, we need to count the number of factors. Imagine writing out 2^{100} using multiplication!

Let's say you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25}, we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: in this case, we can see right away that it is on the 25th day that the number of grains of rice has doubled! That's a lot of rice (more than a cubic meter)!

Lesson 1: Exponent Review

Cool Down: Exponent Check

1. What is the value of 3^4?

2. How many times bigger is 3^{15} compared to 3^{12}?

Unit 7 Lesson 1 Cumulative Practice Problems

1. Write each expression using an exponent:

    a. $1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

    b. $1 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

    c. $1 \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$

    d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

2. Evaluate each expression:

    a. 2^5

    b. 3^3

    c. 4^3

    a. 6^2

    b. $\left(\frac{1}{2}\right)^4$

    c. $\left(\frac{1}{3}\right)^2$

3. Clare made $160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

    a. How much money will Clare have in her account after 1 year? After 2 years?

    b. How much money will Clare have in her account after 5 years? Explain your reasoning.

    c. Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account.

4. The equation $y = 5{,}280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

(From Unit 3, Lesson 1.)

5. The points $(2, 4)$ and $(6, 7)$ lie on a line. What is the slope of the line?

   A. 2

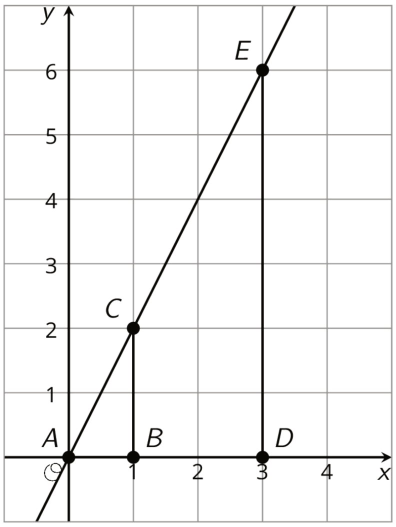
   B. 1

   C. $\frac{4}{3}$

   D. $\frac{3}{4}$

(From Unit 3, Lesson 5.)

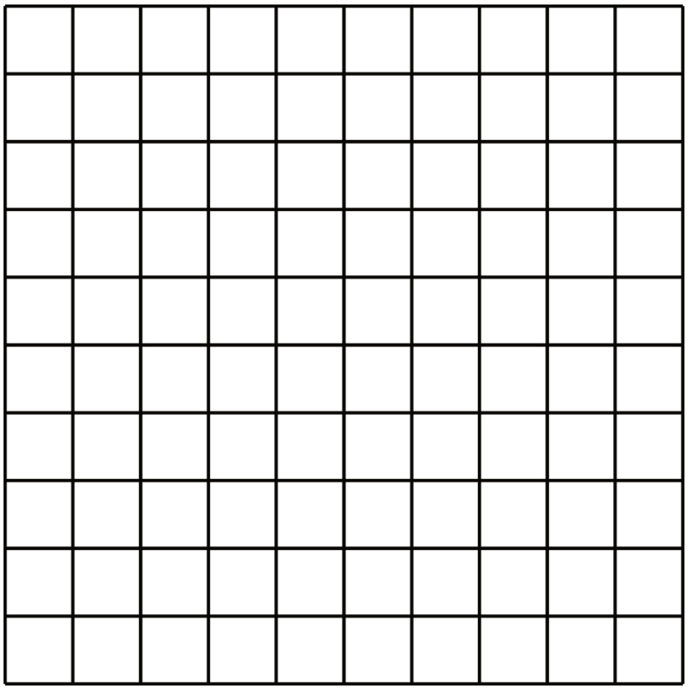
6. The diagram shows a pair of similar figures, one contained in the other. Name a point and a scale factor for a dilation that moves the larger figure to the smaller one.



(From Unit 2, Lesson 6.)

Lesson 2: Multiplying Powers of Ten

2.1: 100, 1, or $\frac{1}{100}$?



Clare said she sees 100.

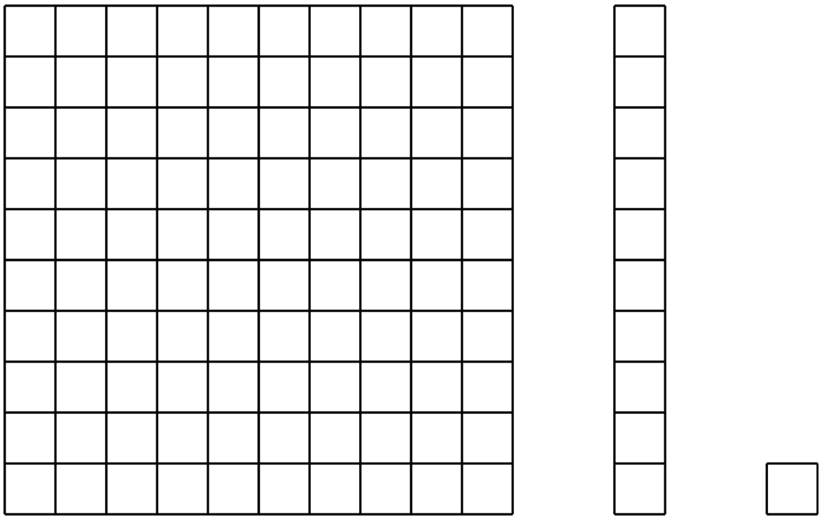
Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Who do you agree with?

2.2: Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could you represent the large square as a power of 10?

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?

4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

2.3: Multiplying Powers of Ten

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Are you ready for more?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?

2. How about 10^7? 10^8?

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: multiplying powers of 10 corresponds to adding the exponents together. To see this, multiply 10^5 and 10^2. We know that 10^5 has five factors that are 10 and 10^2 has two factors that are 10. That means that $10^5 \cdot 10^2$ has 7 factors that are 10.

$$10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7.$$

This will work for other powers of 10 too. So $10^{14} \cdot 10^{47} = 10^{61}$.

This rule makes it easier to understand and work with expressions that have exponents.

Lesson 2: Multiplying Powers of Ten

Cool Down: That's a Lot of Dough, Though!

1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.

2. Each year, roughly 10^6 computer programmers each make about $\$10^5$. How much money is this all together? Express your answer both as a power of 10 and as a dollar amount.

Unit 7 Lesson 2 Cumulative Practice Problems

1. Write each expression with a single exponent:

   a. $10^3 \cdot 10^9$

   b. $10 \cdot 10^4$

   c. $10^{10} \cdot 10^7$

   d. $10^3 \cdot 10^3$

   e. $10^5 \cdot 10^{12}$

   f. $10^6 \cdot 10^6 \cdot 10^6$

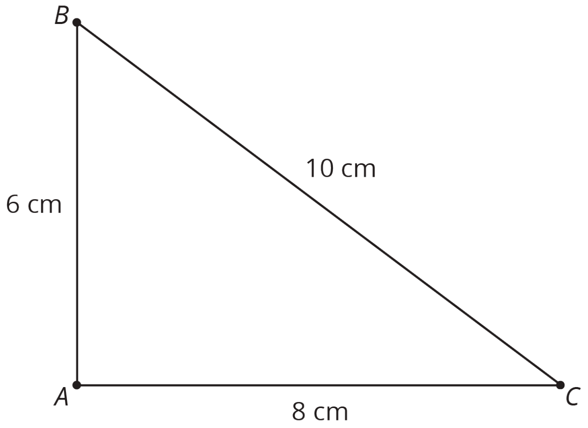
2. A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water.

   a. What is the area of the surface of the water in the pool?

   b. How much water does the pool hold?

   c. Express your answers to the previous two questions as powers of 10.

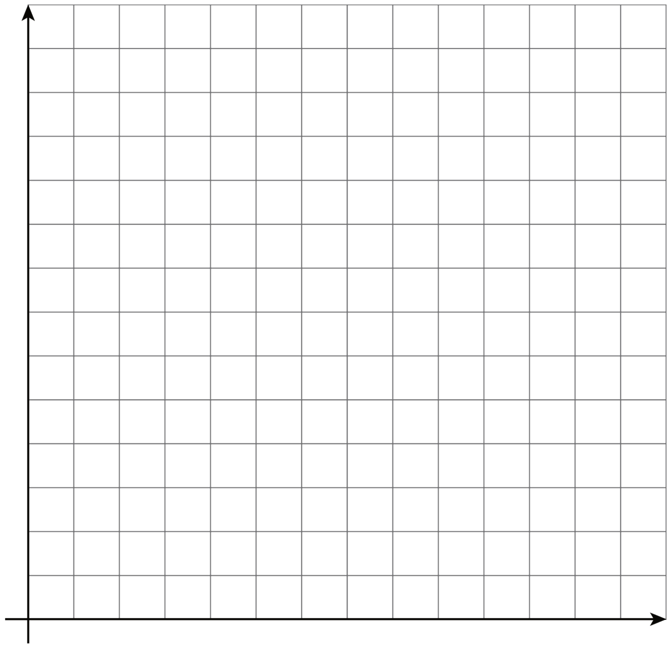
3. Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



(From Unit 2, Lesson 7.)

4. Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

   Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



(From Unit 3, Lesson 3.)

Lesson 3: Powers of Powers of 10

3.1: Big Cube

What is the volume of a giant cube that measures 10,000 km on each side?

3.2: Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

3.3: How Do the Rules Work?

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, “When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$.”
- Elena says, “10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$.”

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Lesson 3 Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$\left(10^4\right)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $\left(10^6\right)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Lesson 3: Powers of Powers of 10

Cool Down: Making a Million

Here are some equivalent ways of writing 10^4:

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents and multiplication.

Unit 7 Lesson 3 Cumulative Practice Problems

1. Write each expression with a single exponent:

    a. $(10^7)^2$

    b. $(10^9)^3$

    c. $(10^6)^3$

    d. $(10^2)^3$

    e. $(10^3)^2$

    f. $(10^5)^7$

2. You have 1,000,000 number cubes, each measuring one inch on a side.

    a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

    b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

    c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

3. An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

   a. How many amoebas are there after 1 hour?

   b. How many amoebas are there after 2 hours?

   c. Write an expression for the number of amoebas after 6 hours.

   d. Write an expression for the number of amoebas after 24 hours.

   e. Why might exponential notation be preferable to answer these questions?

(From Unit 7, Lesson 1.)

4. Elena noticed that, nine years ago, her cousin Katie was twice as old as Elena was then. Then Elena said, “In four years, I’ll be as old as Katie is now!” If Elena is currently e years old and Katie is k years old, which system of equations matches the story?

   A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

   B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

   C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

   D. $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

(From Unit 4, Lesson 15.)

Lesson 4: Dividing Powers of 10

4.1: A Surprising One

What is the value of the expression?

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

4.2: Dividing Powers of Ten

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the “expanded” column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression of the form $10^{\square}$.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Are you ready for more?

expression	expanded	single power
$10^4 \div 10^6$		

4.3: Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. What would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \cdot 10^0$ with a power of 10 with a single exponent using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you multiply 10^{12} by to get this same answer?

2. a. Write $\frac{10^8}{10^0}$ with a single power of 10 using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you divide 10^8 by to get this same answer?

3. If we want the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?

4. Noah says, "If I try to write 10^0 expanded, it should have zero factors that are 10, so it must be equal to 0." Do you agree? Discuss with your partner.

4.4: Making Millions

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents, multiplication, and division. What patterns do you notice with the exponents?

Lesson 4 Summary

In an earlier lesson, we learned that when multiplying powers of 10, the exponents add together. For example, $10^6 \cdot 10^3 = 10^9$ because 6 factors that are 10 multiplied by 3 factors that are 10 makes 9 factors that are 10 all together. We can also think of this multiplication equation as division:

$$10^6 = \frac{10^9}{10^3}$$

So when dividing powers of 10, the exponent in the denominator is subtracted from the exponent in the numerator. This makes sense because

$$\frac{10^9}{10^3} = \frac{10^3 \cdot 10^6}{10^3} = \frac{10^3}{10^3} \cdot 10^6 = 1 \cdot 10^6 = 10^6$$

This rule works for other powers of 10 too. For example, $\frac{10^{56}}{10^{23}} = 10^{33}$ because 23 factors that are 10 in the numerator and in the denominator are used to make 1, leaving 33 factors remaining.

This gives us a new exponent rule:

$$\frac{10^n}{10^m} = 10^{n-m}.$$

So far, this only makes sense when n and m are positive exponents and $n > m$, but we can extend this rule to include a new power of 10, 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means that if we want the rule to work when the exponent is 0, then it must be that

$$10^0 = 1$$

Lesson 4: Dividing Powers of 10

Cool Down: Why Subtract?

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11}? Explain or show your thinking.

Unit 7 Lesson 4 Cumulative Practice Problems

1. Evaluate:

   a. 10^0

   b. $\frac{10^3}{10^3}$

   c. $10^2 + 10^1 + 10^0$

2. Write each expression as a single power of 10.

   a. $\frac{10^3 \cdot 10^4}{10^5}$

   b. $(10^4) \cdot \frac{10^{12}}{10^7}$

   c. $\left(\frac{10^5}{10^3}\right)^4$

   d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$

   e. $\frac{(10^5)^2}{(10^2)^3}$

3. The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

4. Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

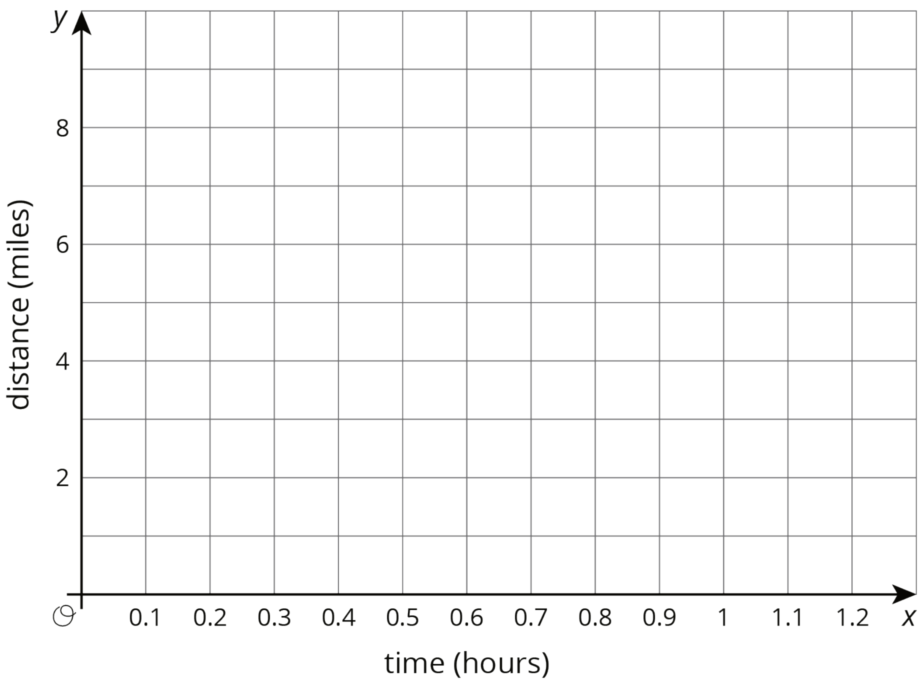
   a. Write an equation relating the two variables.

   b. Rearrange the equation so b is the independent variable.

   c. Rearrange the equation so g is the independent variable.

(From Unit 5, Lesson 3.)

5. Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



(From Unit 3, Lesson 1.)

Lesson 5: Negative Exponents with Powers of 10

5.1: Number Talk: What's That Exponent?

Solve each equation mentally.

$\frac{100}{1} = 10^x$

$\frac{100}{x} = 10^1$

$\frac{x}{100} = 10^0$

$\frac{100}{1,000} = 10^x$

5.2: Negative Exponent Table

Complete the table to explore what negative exponents mean.

$\cdot 10$ $\cdot 10$ $\cdot 10$ $\cdot 10$ $\cdot 10$ $\cdot 10$

using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$

$\cdot\,?$ $\cdot\,?$ $\cdot\,?$ $\cdot\,?$ $\cdot\,?$ $\cdot\,?$

1. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?
2. How does a multiplier of 10 affect the placement of the decimal in the product? How does the other multiplier affect the placement of the decimal in the product?
3. Use the patterns you found in the table to write 10^{-7} as a fraction.
4. Use the patterns you found in the table to write 10^{-5} as a decimal.
5. Write $\frac{1}{100{,}000{,}000}$ using a single exponent.
6. Use the patterns in the table to write 10^{-n} as a fraction.

5.3: Follow the Exponent Rules

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$

$(10^2)^{-3}$

$(10^{-2})^3$

$(10^{-2})^{-3}$

$\frac{1}{(10\cdot10)} \cdot \frac{1}{(10\cdot10)} \cdot \frac{1}{(10\cdot10)}$
$\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)$
$\frac{1}{\frac{1}{10}\cdot\frac{1}{10}} \cdot \frac{1}{\frac{1}{10}\cdot\frac{1}{10}} \cdot \frac{1}{\frac{1}{10}\cdot\frac{1}{10}}$
$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$

   b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$

$\frac{10^2}{10^{-5}}$

$\frac{10^{-2}}{10^5}$

$\frac{10^{-2}}{10^{-5}}$

$\frac{\frac{1}{10}\cdot\frac{1}{10}}{\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}}$
$\frac{10\cdot10}{10\cdot10\cdot10\cdot10\cdot10}$
$\frac{\frac{1}{10}\cdot\frac{1}{10}}{10\cdot10\cdot10\cdot10\cdot10}$
$\frac{10\cdot10}{\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}\cdot\frac{1}{10}}$

   b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$10^4 \cdot 10^{-3}$

$10^{-4} \cdot 10^3$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10)$
$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Are you ready for more?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, “I knew I’d be the only one left, since I went first.”

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?

2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Lesson 5 Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1:

$$10^8 \cdot \frac{1}{10} = 10^7$$

This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2:

$$\left(\frac{1}{10}\right)^2 \cdot 10^8 = 10^6$$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make

$$10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents:

$$\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

To see why, notice that

$$\frac{10^3}{10^5} = \frac{10^3}{10^3 \cdot 10^2} = \frac{10^3}{10^3} \cdot \frac{1}{10^2} = \frac{1}{10^2}$$

which is equal to 10^{-2}.

Here is an example of extending the rule $(10^m)^n = 10^{m \cdot n}$ to use negative exponents:

$$\left(10^{-2}\right)^3 = 10^{(-2)(3)} = 10^{-6}$$

To see why, notice that $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. This means that

$$\left(10^{-2}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$$

Lesson 5: Negative Exponents with Powers of 10

Cool Down: Negative Exponent True or False

Mark each of the following equations as true or false. Explain or show your reasoning.

1. $10^{-5} = -10^{5}$
2. $(10^{2})^{-3} = (10^{-2})^{3}$
3. $\frac{10^{3}}{10^{14}} = 10^{-11}$

Unit 7 Lesson 5 Cumulative Practice Problems

1. Write with a single exponent: (ex: $\frac{1}{10} \cdot \frac{1}{10} = 10^{-2}$)

   a. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   b. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   c. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^2$

   d. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^3$

   e. $(10 \cdot 10 \cdot 10)^{-2}$

2. Write each expression as a single power of 10.

   a. $10^{-3} \cdot 10^{-2}$

   b. $10^4 \cdot 10^{-1}$

   c. $\frac{10^5}{10^7}$

   d. $(10^{-4})^5$

   e. $10^{-3} \cdot 10^2$

   f. $\frac{10^{-9}}{10^5}$

3. Select **all** of the following that are equivalent to $\frac{1}{10{,}000}$:

   A. $(10{,}000)^{-1}$

   B. $(-10{,}000)$

   C. $(100)^{-2}$

   D. $(10)^{-4}$

   E. $(-10)^2$

4. Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

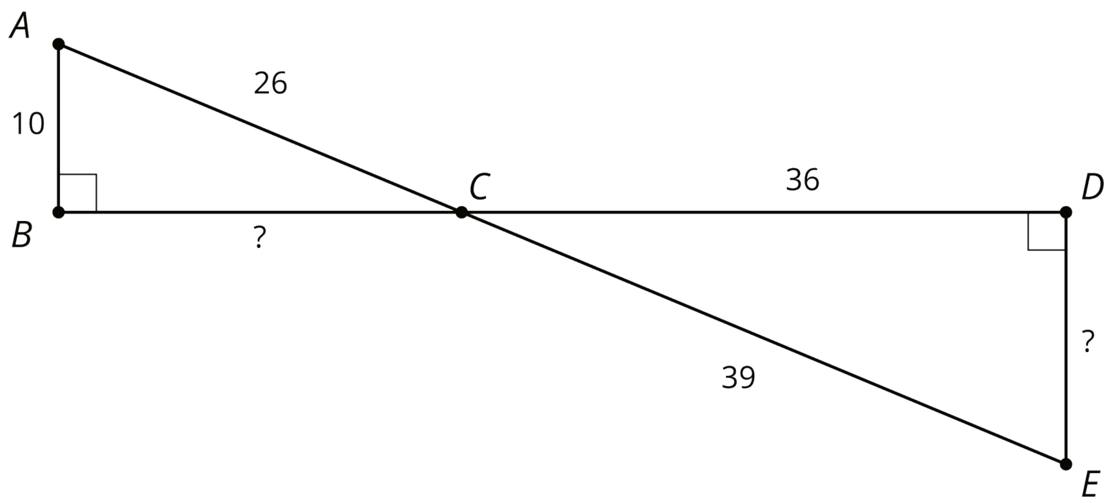
d. $y = \frac{5}{2}x$

Situations:

- A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- A store has a "4 for $10" sale on hats.
- For every 48 cookies I bake, my students get 24.

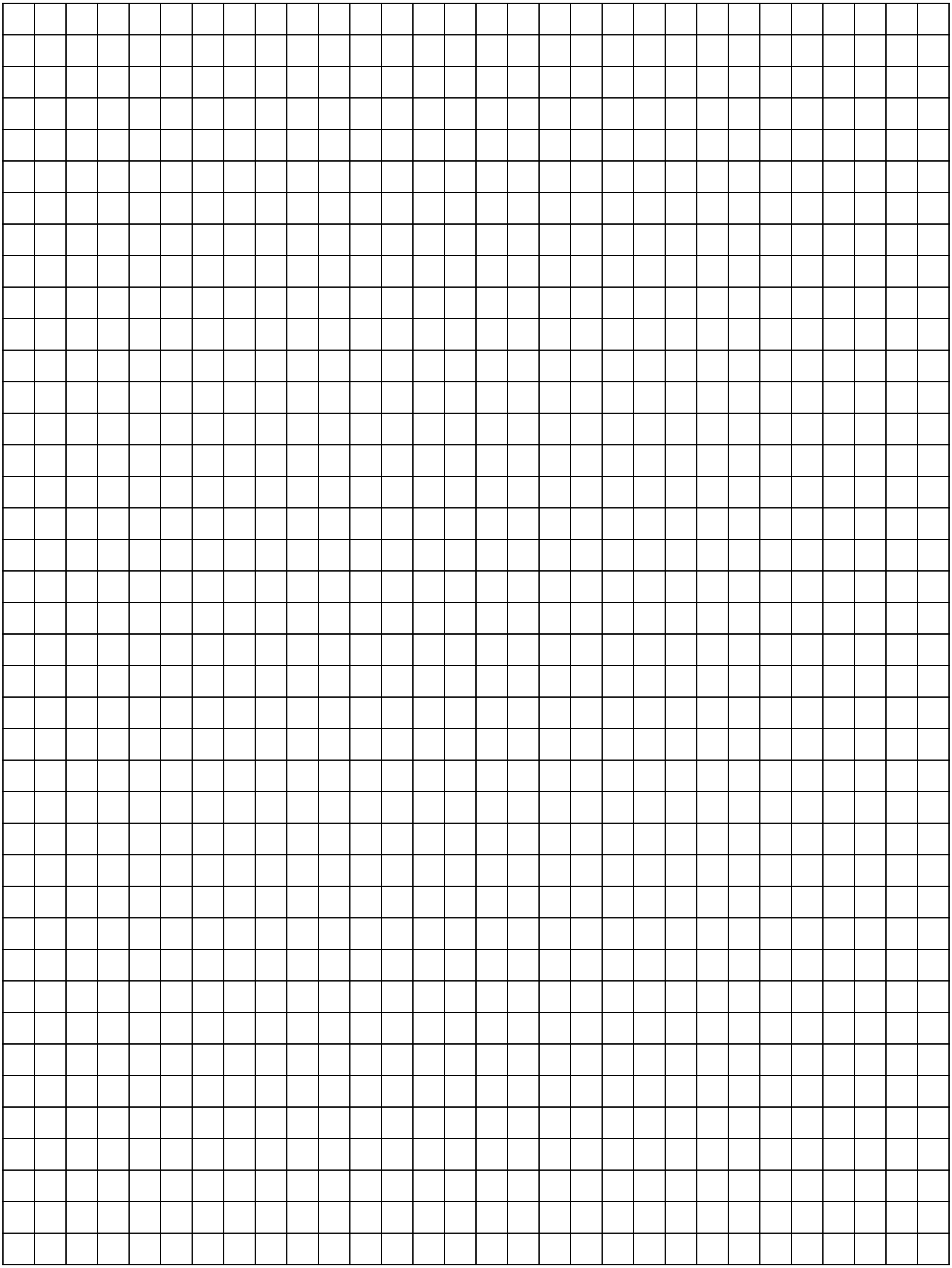
(From Unit 3, Lesson 2.)

5. a. Explain why triangle ABC is similar to EDC.



b. Find the missing side lengths.

(From Unit 2, Lesson 8.)



Family Support Materials

Exponents and Scientific Notation

Exponent Review

Family Support Materials 1

This week your student will learn the rules for multiplying and dividing expressions with exponents. Exponents are a way of keeping track of how many times a number has been repeatedly multiplied. For example, instead of writing $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$, we can write 8^7 instead. The number repeatedly multiplied is called the base, which in this example is 8. The 7 here is called the exponent.

Using our understanding of repeated multiplication, we'll figure out several "rules" for exponents. For example, suppose we want to understand the expression $10^3 \cdot 10^4$. Rewriting this to show all the factors, we get $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$. Since this is really 7 10s multiplied together, we can write $10^3 \cdot 10^4 = 10^7$. By counting the repeated factors that are 10, we've added the exponents together (there are 3 of them, and then 4 more). This leads us to understanding a more general rule about exponents; when multiplying powers of the same base, we add the exponents together:

$$x^n \cdot x^m = x^{n+m}$$

Using similar reasoning, we can figure out that when working with powers of powers, we multiply the exponents together:

$$(x^n)^m = x^{n \cdot m}$$

These patterns will lead to other discoveries later on.

Here is a task to try with your student:

1. Jada and Noah were trying to understand the expression $10^4 \cdot 10^5$. Noah said, "since we are multiplying, we will get 10^{20}." Jada said, "But I don't think you can get 20 10s multiplied together from that." Do you agree with either of them?

2. Next, Jada and Noah were thinking about a similar expression, $\left(10^4\right)^5$. Noah said, "Ok this one will be 10^{20} because you will have 5 groups of 4." Jada said, "I agree it will be 10^{20}, but it's because there will be 4 groups of 5." Do you agree with either of them?

Solution:

1. Jada is correct. Rewriting $10^4 \cdot 10^5$ to show all the factors looks like $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$. We can see that there are a total of 9 10s being multiplied. This helps us understand what's going on when we use the rule to write $10^4 \cdot 10^5 = 10^{4+5} = 10^9$.

2. This time, Noah is correct. When we look at $\left(10^4\right)^5$, the outside exponent of 5 tells us that there are 5 10^4s being multiplied together. So $\left(10^4\right)^5 = 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4$. This means there are 5 groups of 4 10s being multiplied together. We could write this out the long way as $\left(10^4\right)^5 = (10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$. This helps us understand what's going on when we use the rule to write $\left(10^4\right)^5 = 10^{4 \cdot 5} = 10^{20}$.

Scientific Notation

Family Support Materials 2

This week your student will use powers of 10 to work with very large or very small numbers. For example, the United States mint has made over 500,000,000,000 pennies. In order to understand this number, we have to count all of the zeros. Since there are 11 of them this means there are 500 billion pennies. Using powers of 10, we can write this as $5 \cdot 10^{11}$. The advantage to this way of writing the number is that we can see right away how many zeros there are (11), and more efficiently compare numbers when they are both written in this form. The same is true for small quantities. For example, a single atom of carbon weighs about 0.0000000000000000000000199 grams. If we write this using powers of 10, it becomes $(1.99) \cdot 10^{-23}$.

Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to add or take away a zero when writing out the decimal without realizing! Writing numbers in this way is called scientific notation. We can use the exponent rules learned earlier to estimate and solve problems with scientific notation.

Here is a task to try with your student:

This table shows the top speeds of different vehicles.

vehicle	speed (kilometers per hour)
sports car	$(4.15) \cdot 10^2$
Apollo Command/Service Module	$(3.99) \cdot 10^4$
jet boat	$(5.1) \cdot 10^2$
autonomous drone	$(2.1) \cdot 10^4$

1. Order the vehicles from fastest to slowest.
2. The top speed of a rocket sled is 10,326 kilometers per hour. Is this faster or slower than the autonomous drone?
3. Estimate how many times as fast the Apollo Command/Service Module is than the sports car.

Solutions

1. The order is: Apollo CSM, autonomous drone, jet boat, sports car. Since all of these values are in scientific notation, we can look at the power of 10 to compare. The speeds of the Apollo CSM and autonomous drone both have the highest power of 10 (10^4), so they are fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because their speeds both have the same power of 10 (10^2) but 5.1 is greater than 4.15.

2. The autonomous drone is faster than the rocket sled. In scientific notation, the rocket sled's speed is $1.0326 \cdot 10^4$, and the drone's speed is $2.1 \cdot 10^4$ and 2.1 is greater than 1.0326.

3. To find how many times as fast the Apollo CSM is than the sports car, we are trying to find out what number times $4.15 \cdot 10^2$ equals $3.99 \cdot 10^4$. So we are trying to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$. Using exponent rules and our understanding of fractions, we have $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^{4-2} = 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!